Black Holes, Entanglement and Random Matrices

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When is entanglement captured by semiclassical geometry?

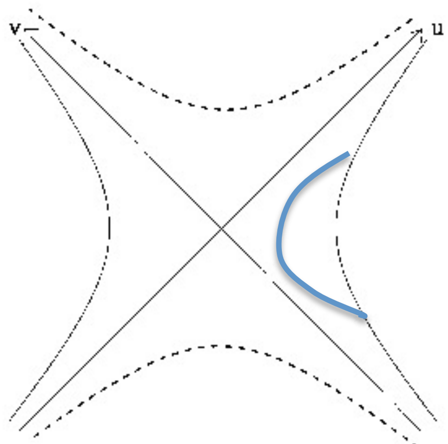
- Y – Always
- M - Sometimes (depending on the state, depending on the probe,…)
- N - Never
- (Who's asking?)

Can we provide a quantitative criteria given:

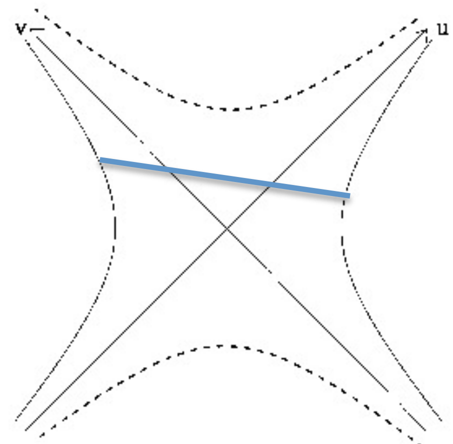
1. The probe (what is the minimal information on the probe).
2. The state (what is the minimal information on the state).

QM toy models => AdS/CFT models

Basic criteria:



G_{RR}



G_{LR}

1. $G_{RR} \approx G_{LR}$ (no entropy suppression)

2. G >> STD(G)

Not quite enough: existence of wormhole depends on the spectral properties of the probe.

Eternal BH vs. Thermal AdS

$$|\psi_\beta\rangle = \frac{1}{Z(\beta)} \sum_i e^{-\beta E_i/2} |i\rangle_L \otimes |i\rangle_R,$$

Wormhole:

High temperature phase of AdS black hole

S>>1 strongly coupled plasma.

No wormhole:

- Begin with energy below the HP phase transition.
- Keep in the microcanonical ensemble, and pump energy in.

S>>1, free field (bulk)

Random operators vs. structured

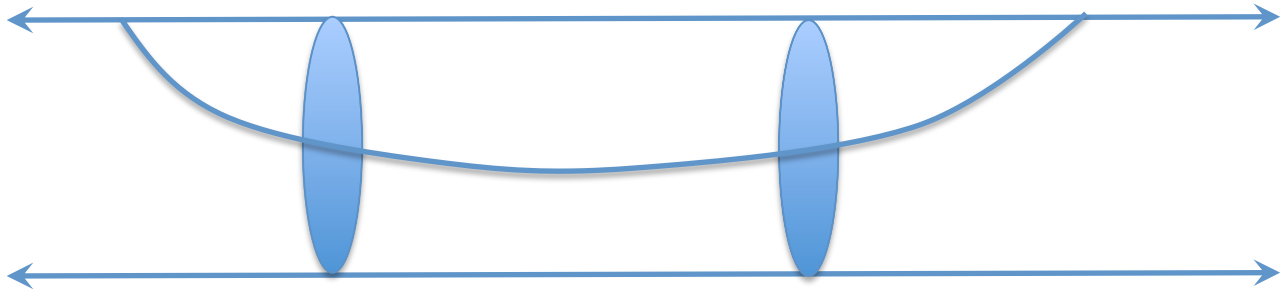
$$< \vec{n}, n_i | T^i | \vec{m}, m_i >= \delta_{\vec{n},\vec{m}} \big(\delta_{n_i+1, m_i} + \delta_{n_i, m_i+1} \big)$$

$$\langle T_1^{(1)}(t) T_1^{(1)}(0) \rangle = e^{-iw_1 t} + e^{iw_1 t - \beta w_1},$$

$$\langle T_1^{(2)}(t) T_1^{(1)}(0) \rangle = \big(e^{-iw_1 t} + e^{iw_1 t} \big) e^{-\beta w_1/2}.$$

No entropy suppression in LR vs. RR. But still no wormhole.

There is a semiclassical solution – a Euclidean AdS instanton which tunnels into two Mink. AdS.



A ER bridge can exist only in a case of a

1) The probe is a random matrix
2) The Hamiltonian is a random matrix.

Fixed energy ensemble

H_E states with energies between (E-Δ, E+Δ)

Entangled state in $H_{E,R}$*$H_{E,L}$, i.e.

$$|\psi_\beta\rangle = \frac{1}{Z(\beta)} \sum_i e^{-\beta E_i/2} |i\rangle_L \otimes |i\rangle_R,$$

$$|\psi_{\mathrm{U}}\rangle = \frac{1}{d_E} \sum_{i,j \in \mathcal{H}_E} U_{ij} |i\rangle_L \otimes |j\rangle_R$$

$$|\psi_c\rangle = \sum_{i,j} c_{ij} |i, j\rangle \quad \text{with} \quad \sum_{i,j} |c_{ij}|^2 = 1$$

Structured vs. unstructured pieces of the operator

Denote the probe operator by O. We would like to study how it acts on states which make the black hole – say some ensemble of states with at a given energy. The states of the black hole contain some unspecified information about the state "behind the horizon" and also about particles outside the horizon. We are interested in how the operator acts on the former degrees of freedom.

In the Eikonal approximation we are interested in geodesic which graze the horizon, or go through it.

Also in Eikonal, for such geodesics, if there is a semiclassical description and no firewall, we can expect

$G \approx e^{-ml}$

If there is some benign firewall then maybe

$G \approx A * e^{-ml}$, A -> as the firewall becomes less benign.

However, right now we only have a toy model for such correlators in QM. We need to be able to

1) Incoroprate conformal invariance, which will give us access to different m's.
2) Incorporate large-N limit, to have a semiclassical space to start with.

Both seem to be doable.

The unstructured piece of O is a random matrix, M_{ij} taken from some distribution (in the basis of energy eigenstates with dense spectrum)

$$\mathcal{F}_r = \frac{1}{Z_{\mathrm{r}}^{M}} dM_{ij} dM_{ij}^{*}\, e^{-\gamma \mathrm{tr}\left(M\, M^{\dagger}\right)},$$

Invariance under $U(e^S)$ ⇔ maximal ignorance/difficulty assumption

$$\mathbf{E}\left(M_{ij}^{*} M_{kl}\right) = \frac{1}{\gamma} \delta_{ik} \delta_{jl}\,.$$

Determination of the normalization via "finite total cross section"

$$\mathbf{E}\left(\sum_{k \in \mathcal{H}_E} |\langle k|\mathcal{O}|i_0\rangle|^2\right) = \frac{e^{S(E)}}{\gamma}, \quad \rightarrow \quad \gamma = \hat{\gamma} e^{S(E)},$$

Single sided correlator

$$\mathbf{E}\big(\langle\psi_{\mathrm{U}}|\mathcal{O}_R^\dagger(0)\mathcal{O}_R(0)|\psi_{\mathrm{U}}\rangle\big) = e^{-S} \sum_{i\in\mathcal{H}_E, n\in\mathcal{H}_E} \mathbf{E}\big(|\langle n|\mathcal{O}_R|i\rangle|^2\big) = e^{-S}\,\frac{e^{2S}}{\gamma} = \frac{1}{\hat{\gamma}}\,.$$

$$\mathbf{E}\big(\langle\psi_{\mathrm{U}}|\mathcal{O}_R^\dagger(t)\mathcal{O}_R(0)|\psi_{\mathrm{U}}\rangle\big) = \frac{e^{-S}}{\gamma}\sum_{i,n} e^{i(E_i-E_n)t} = \frac{e^{-2S}}{\hat{\gamma}}\big|\mathrm{tr}_{\mathcal{H}_E}(W(t))\big|^2$$

Two sided correlator

$$\mathcal{O}_L(t) = \tilde{\mathcal{O}}_L(-t) = \mathcal{O}_R(-t)^T\,.$$

$$\begin{aligned}\mathbf{E}\big(\langle\psi_{\mathrm{U}}|\mathcal{O}_R^\dagger(t)\mathcal{O}_L(0)|\psi_{\mathrm{U}}\rangle\big) &= e^{-S(E)}\sum_{i,j,k,l} U_{ij}\,U_{kl}^\star\,e^{i(E_l-E_j)t}\,\mathbf{E}\big(M_{jl}^\star\,M_{ik}\big)\\ &= \frac{e^{-2S(E)}}{\hat{\gamma}}|\mathrm{tr}\,(U\,W(t))\,|^2,\end{aligned}$$

Equal time single sided correlator $\frac{1}{\hat{\gamma}}$

Equal time two sided correlator $\frac{1}{\hat{\gamma}} e^{-2S} \left| tr(U) \right|^2$

$$\mathbf{E}_{\mathrm{U}}\big(\mathbf{E}\big(\langle\psi_{\mathrm{U}}|\mathcal{O}_R^\dagger(0)\mathcal{O}_L(0)|\psi_{\mathrm{U}}\rangle\big)\big) = \frac{1}{d_E^2\,\hat{\gamma}} \sum_{i,j} \int dU U_{ii} U_{jj}^* = \frac{1}{d_E^3\,\hat{\gamma}} \sum_{i,j} \delta_{ij} = \frac{1}{d_E^2\,\hat{\gamma}}\,,$$

Standard deviations:

$$\sigma^2_{\mathcal{O},RR}(\mathrm{U},t) = \mathbf{E}\big(|\langle\psi_{\mathrm{U}}|O_R^\dagger O_R|\psi_{\mathrm{U}}\rangle|^2\big) - |\mathbf{E}\big(|\langle\psi_{\mathrm{U}}|O_R^\dagger O_R|\psi_{\mathrm{U}}\rangle\big)|^2,$$
$$\sigma^2_{\mathcal{O},RL}(\mathrm{U},t) = \mathbf{E}\big(|\langle\psi_{\mathrm{U}}|O_R^\dagger O_L|\psi_{\mathrm{U}}\rangle|^2\big) - |\mathbf{E}\big(|\langle\psi_{\mathrm{U}}|O_R^\dagger O_L|\psi_{\mathrm{U}}\rangle\big)|^2.$$

$$\sigma^2_{\mathcal{O},RR}(\mathrm{U},t) = \frac{e^{-2S(E)}}{\hat{\gamma}^2}. \quad \sigma^2_{\mathcal{O},RL}(\mathrm{U},t) = \frac{e^{-2S(E)}}{\hat{\gamma}^2}.$$

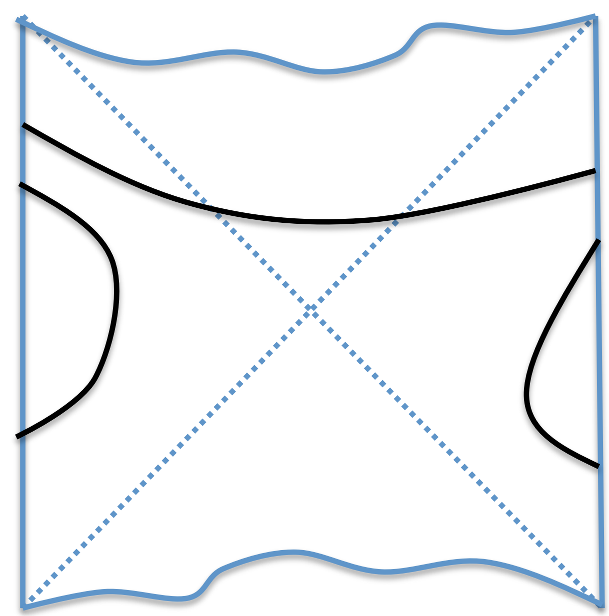
$$\frac{\sigma^2_{\mathcal{O},RR}(\mathrm{U},t)}{|\mathbf{E}\big(|\langle\psi_{\mathrm{U}}|O_R^\dagger O_R|\psi_{\mathrm{U}}\rangle\big)|^2} = \frac{e^{2S}}{|\mathrm{tr}W(t)|^4},$$
$$\frac{\sigma^2_{\mathcal{O},RL}(\mathrm{U},t)}{|\mathbf{E}\big(|\langle\psi_{\mathrm{U}}|O_R^\dagger O_L|\psi_{\mathrm{U}}\rangle\big)|^2} = \frac{e^{2S}}{|\mathrm{tr}\,\mathrm{U}W(t)|^4}.$$

Perturbation theory?

$$\left\langle M_L \, M_R M_L \ldots M_R M_R \, M_L \right\rangle$$

$$\mathbf{E}\left(M_{ij} M^{\star}_{kl}\right) = \frac{\delta_{ik}\delta_{jl}}{\gamma\,\Delta_{ij}}\,,$$ rather

then a Wick contraction



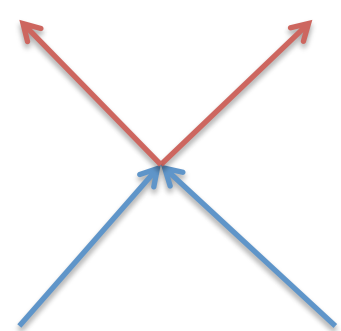
Interactions:

$$\left\langle M_L G_R M_L \ldots M_R G_R M_L \right\rangle$$

Interactions can be encoded in joint distribution of the random matrices on the BH states

$$dMdM^* dGdG^* * \exp\left[-e^S tr(MM^{\perp}) - e^S tr(GG^{\perp}) - \alpha \cdot e^S tr(MG^{\perp}M^{\perp}G) - \alpha \cdot e^S tr(MGM^{\perp}G^{\perp}) + \ldots\ldots\right]$$

which can give rise to a vertex

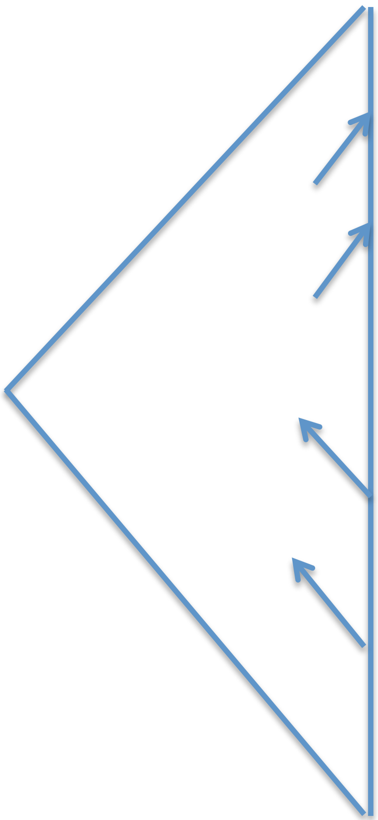


Not perturbation theory?

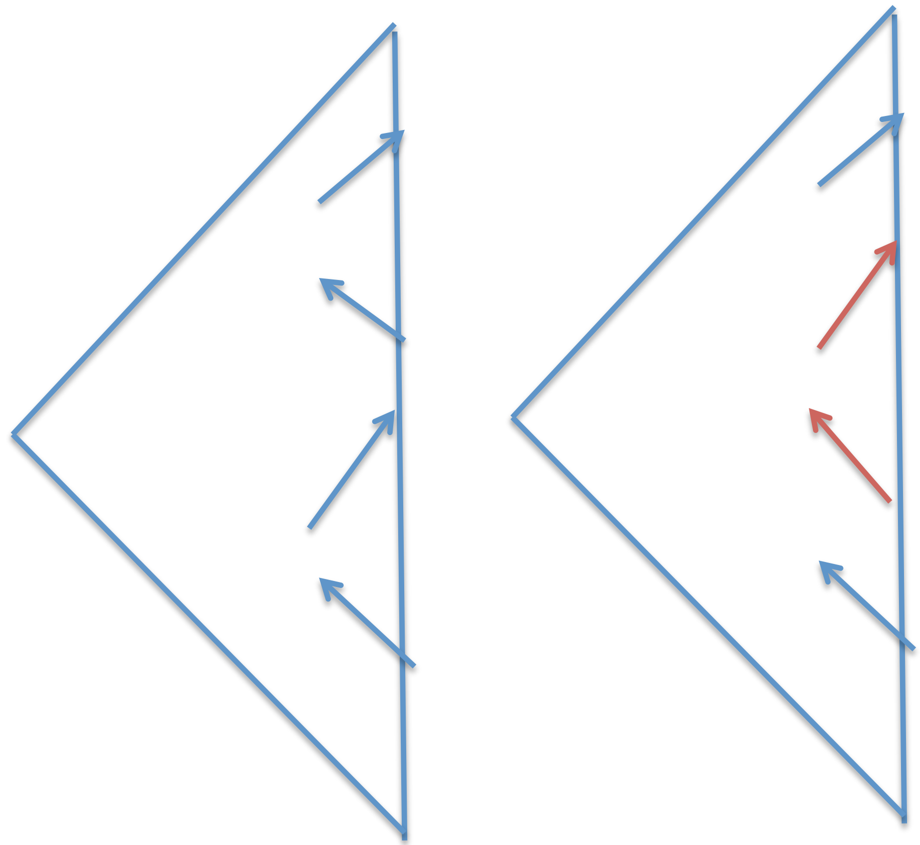
$$C_1 = \mathbf{E}\left(M_2^\dagger M_1^\dagger M_2 M_1\right) \ , \qquad C_2 = \mathbf{E}\left(M_2^\dagger M_2 M_1^\dagger M_1\right) \ ,$$

1) The M's are field operators which insert a particle/ extract an anti-particle.
2) All the M's are the same operator, the indices just denote different time insertions.

$$C_1 = \mathbf{E}\left(M_2^\dagger M_1^\dagger M_2 M_1\right)\ , \qquad C_2 = \mathbf{E}\left(M_2^\dagger M_2 M_1^\dagger M_1\right)\ ,$$



Two ways of contracting



In an ordinary Wick contraction, there are two contractions in each protocol, with similar strength.

In the large random matrix computation there is a planarity restriction

C2:

$$\mathbf{E}\left(M^*_{k_1 i} M_{k_1 k_2}\right) \mathbf{E}\left(M^*_{k_3 k_2} M_{k_3 i}\right) = \delta_{k_1 k_1} \delta_{i k_2} \delta_{k_3 k_3} \delta_{k_2 i} = d_E^3 \,.$$

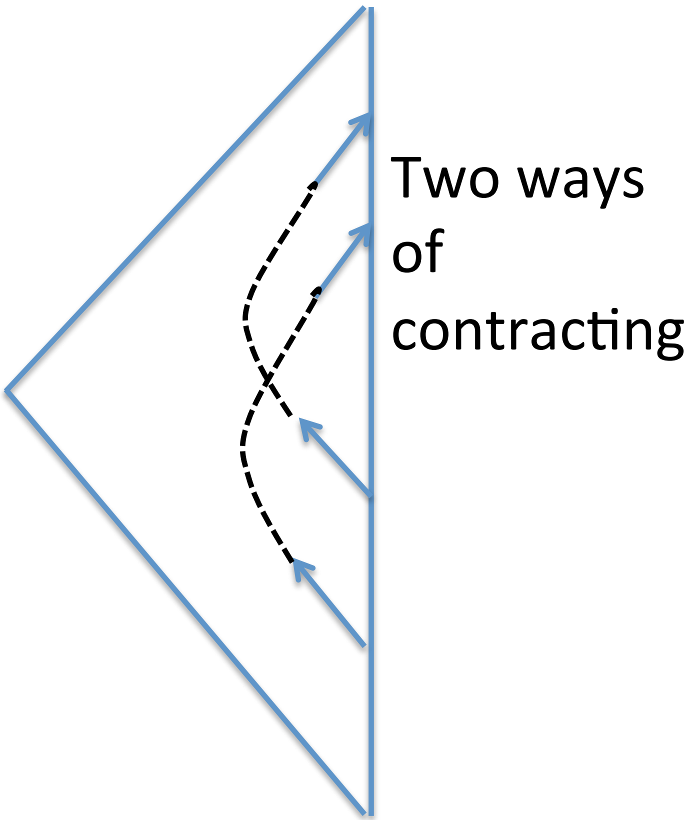
$$\mathbf{E}\left(M^*_{k_1 i} M_{k_3 i}\right) \mathbf{E}\left(M^*_{k_3 k_2} M_{k_1 k_2}\right) = \delta_{k_1 k_3} \delta_{i i} \delta_{k_1 k_3} \delta_{k_2 k_2} = d_E^3 \,.$$

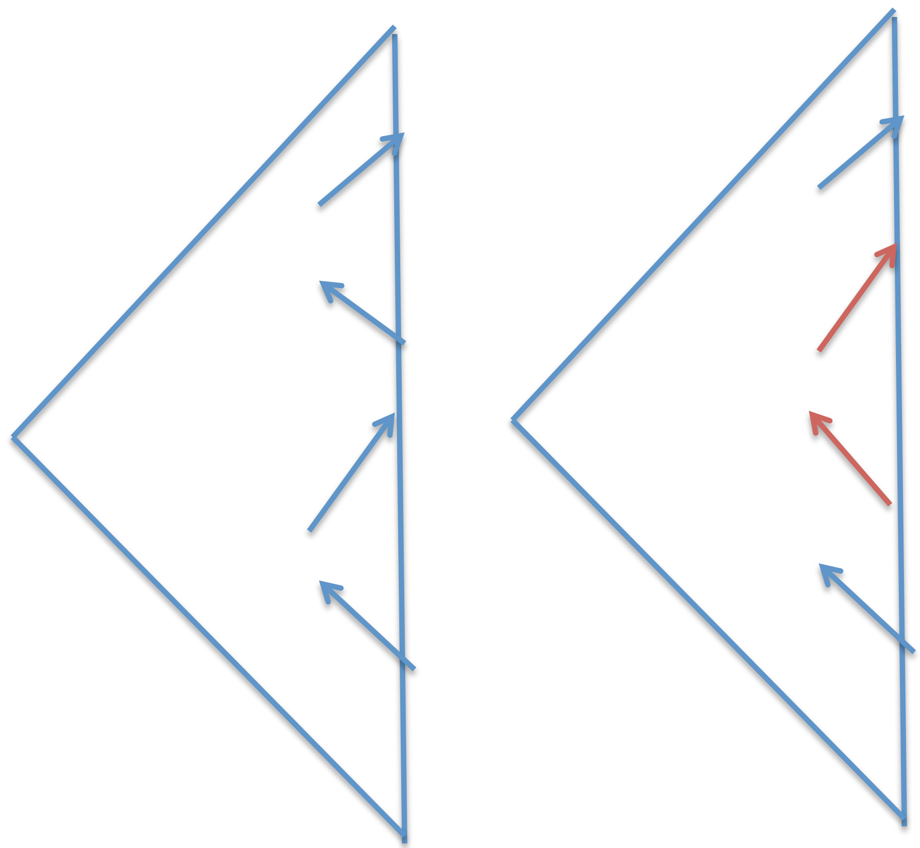
C1:

$$\mathbf{E}\left(M^*_{k_1 i} M_{k_3 i}\right) \mathbf{E}\left(M^*_{k_2 k_1} M_{k_2 k_3}\right) = \delta_{k_1 k_3} \delta_{i i} \delta_{k_2 k_2} \delta_{k_1 k_3} = d_E^3$$

$$\mathbf{E}\left(M^*_{k_1 i} M_{k_2 k_3}\right) \mathbf{E}\left(M^*_{k_2 k_1} M_{k_3 i}\right) = \delta_{k_1 k_2} \delta^{'}_{i k_3} \delta_{k_2 k_3} \delta_{k_1 i} \overset{'}{=} d_E \,,$$

$$C_1 = \mathbf{E}\left(M_2^\dagger M_1^\dagger M_2 M_1\right) \; , \qquad C_2 = \mathbf{E}\left(M_2^\dagger M_2 M_1^\dagger M_1\right) \; ,$$





One contraction is excluded by planarity. The kinematics corresponds to the intersection of geodesics where one is infalling and one is outgoing, very close to the horizon.

Energy changing ensemble

$$\mathcal{F}_{\mathrm{g}} = \frac{1}{Z_{\mathrm{g}}^{M}} \prod_{ij} dM_{ij} dM_{ij}^{*} \, e^{-\gamma \left(\mathrm{tr}(MM^{\dagger}) - \alpha_1 \mathrm{tr}([M,H]M^{\dagger}) + \alpha_2 \mathrm{tr}([H,M][M^{\dagger},H]) + \ldots \right)},$$

Computations are similar

$$\mathcal{F}_{\mathrm{g}} \propto \Pi_{ij} dM_{ij} dM_{ij}^{*} e^{-\gamma \sum_{kl} \Delta_{kl} |M_{kl}|^2},$$

$$\Delta_{kl} = 1 + \alpha_1 (E_k - E_l) + \alpha_2 (E_k - E_l)^2 + \cdots \equiv P(\alpha_j, E_k - E_l),$$

$$\gamma = \hat{\gamma} \left(e^{S(E_i)} + e^{S(E_j)} \right).$$

$$\frac{1}{Z(\beta)}\mathbf{E}\left(\mathrm{tr}(e^{-\beta H}\mathcal{O}_R^\dagger(t)\mathcal{O}_R(0))\right) = \frac{1}{Z(\beta)}\sum_{ik} e^{-\beta E_i}\, e^{i(E_i-E_k)t}\,\mathbf{E}\left(M_{ki}^* M_{ki}\right)$$
$$= \frac{1}{Z(\beta)}\int dE_i dE_k \frac{e^{-\beta E_i+S(E_i)+S(E_k)+i(E_i-E_k)t}}{\hat{\gamma}\left(e^{S(E_i)}+e^{S(E_k)}\right)P(\alpha_j, E_k-E_i)}.$$

We can expand in E_k-E_i and obtain, up to 1/extensive,

$$\int d\Delta \frac{e^{-it\Delta}}{\hat{\gamma}(1+e^{-\beta\Delta})\,P(\alpha_l,\Delta)}.$$

At large t, we can localize to the pole in P (with the smallest imaginary part) and obtain an exponential decay of the correlator. I.e. Pole of P ⇔ quasi normal modes.

Single and two sided correlator $|\psi_c\rangle = \sum_{i,j \in \mathcal{H}} c_{ij} |i, j\rangle\,,$

Single sided:

$$\int dE_k \sum_i (c^\dagger c)_{ii} \frac{e^{S(E_k)+i(E_i-E_k)t}}{\hat{\gamma}(e^{S(E_i)}+e^{S(E_k)})(1+\alpha_1(E_k-E_i)+\alpha_2(E_k-E_i)^2)}$$

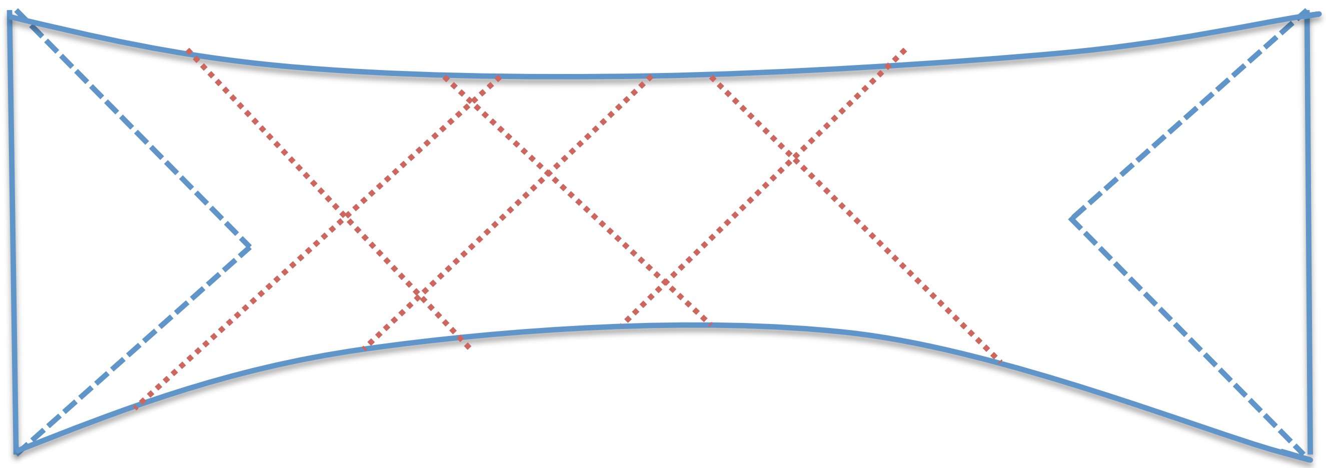
Two sided correlator

$$\mathbf{E}\big(\langle\psi_c|\mathcal{O}_R^\dagger(t)\mathcal{O}_L(0)|\psi_c\rangle\big) = \sum_{i,k} c_{ii} c_{kk}^* \frac{e^{i(E_k-E_i)t}}{\gamma\,\Delta_{ik}}\,.$$

$$= \frac{1}{Z}\int dE_i dE_k F(E_i) F^*(E_k) \frac{e^{-\beta E_i/2-\beta E_k/2+S(E_i)+S(E_k)}}{e^{S(E_i)}+e^{S(E_k)}} \frac{e^{i(E_k-E_i)t}}{\hat{\gamma}\Delta_{ik}(E_i,E_k)}$$

where $F(E) = e^{-S(E)} \sum_{i \in \mathcal{H}} U_{ii},$

Shenker-Stanford configurations: Length and randomness



$$e^{-ml_{shocks}} << e^{-ml_0}$$

Recall that G ≈ e^{-2S} $|tr(U)|^2 \approx e^{-ml}$

so when U goes away from the identity, the length increases. For U which is close to identity U ≈ $e^{i\alpha G}$. G fixed, α taken to zero. This will increase l by a little (this does not work by itself. There is some kinematic dressing that one needs to do).

The state is generated by shock waves made by some operator acting on the boundaries. Such operators are random operators. So G can be taken from a random ensemble

$$dGdG^{\perp} * \exp\left[-e^{S} tr(GG^{\perp})\right]$$

and we can evaluate

$$V_{\alpha} = E_{G}\left(\left|tr(e^{i\alpha G})\right|^{2}\right), \quad \hat{V}_{\alpha} = e^{-2S} V_{\alpha}$$

Consider concatenating two shocks

$$V_{\alpha_1\alpha_2} = |\mathrm{tr}\left(e^{i\alpha_1 H_1} e^{i\alpha_2 H_2}\right)|^2$$

$$\int dU \, \mathrm{tr}(AUBU^\dagger)\mathrm{tr}(UB^\dagger U^\dagger A^\dagger), \quad A = e^{i\alpha_1 H_1}, \; B = e^{i\alpha_2 H_2}$$

$$= \frac{1}{N^2-1}\left(\mathrm{tr}(A)\mathrm{tr}(A^\dagger)\mathrm{tr}(B)\mathrm{tr}(B^\dagger) + \mathrm{tr}(AA^\dagger)\mathrm{tr}(BB^\dagger)\right)$$
$$- \frac{1}{N(N^2-1)}\left(\mathrm{tr}(A)\mathrm{tr}(A^\dagger)\mathrm{tr}(BB^\dagger) + \mathrm{tr}(AA^\dagger)\mathrm{tr}(B)\mathrm{tr}(B^\dagger)\right)$$

$\Rightarrow \quad \hat{V}_{\alpha_1\alpha_2} = \hat{V}_{\alpha_1}\hat{V}_{\alpha_2}$