Entanglement entropy on the fuzzy sphere

(arXiv:1310.8345 + arXiv:1409.XXXX)

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Gong Show

Motivation

[1] Fischler, Kundu and Kundu '13, Karczmarek and Rabideau '13

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- Holographic entanglement entropy for noncommutative theories does not follow the *area law* [1]: can we see this from a *field theory* point of view?

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- The fuzzy sphere has a strange *distribution of DOF*: how does this affect EE and mutual information?

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- The fuzzy sphere has a strange *distribution of DOF*: how does this affect EE and mutual information?
- To what extent is the fuzzy sphere *more than just a regularization* of the sphere?

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The fuzzy sphere

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- *Non-commutative* coordinates:
  $x_a \to \hat{X}_a = R\frac{L_a}{\sqrt{j(j+1)}} \quad , \quad N = 2j+1$
  $[L_a, L_b] = i\epsilon_{abc}L_c$

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  $\Rightarrow H = \frac{1}{2}Tr\left\{(\partial_t\hat{\Phi})^2 - [L_a, \hat{\Phi}][L_a, \hat{\Phi}] + \hat{\Phi}\mu^2\hat{\Phi}\right\}$

The fuzzy sphere

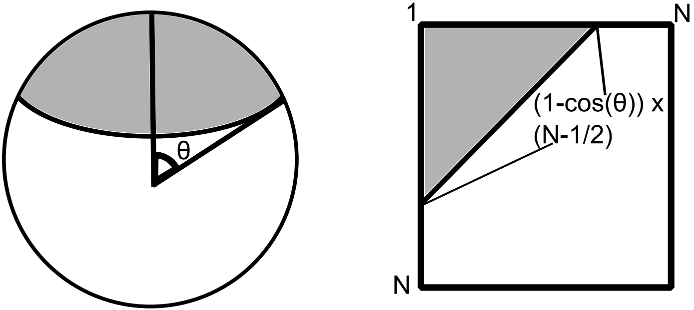
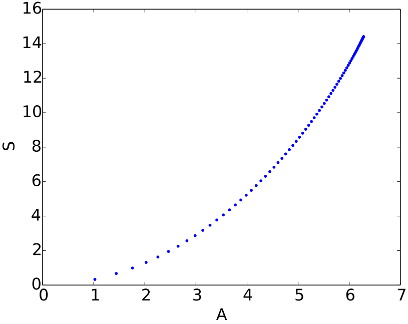
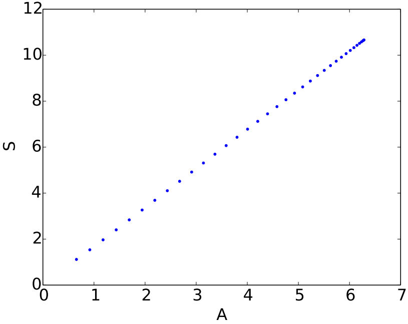


Figure : Degrees of freedom on the fuzzy sphere and their matrix counterparts

Results: Entanglement Entropy



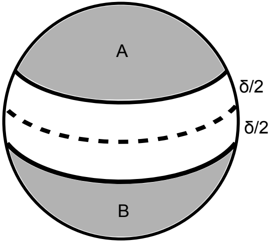
(a) Fuzzy sphere



(b) Commuative sphere

Figure : Entanglement entropy vs. area of boundary, $\mu = 1.0$.

Results: Mutual Information



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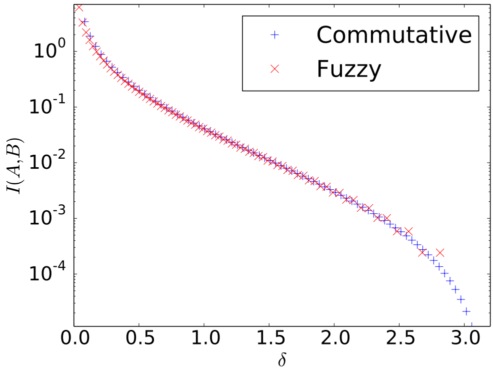
$S_A + S_B - S_{A \cup B}$



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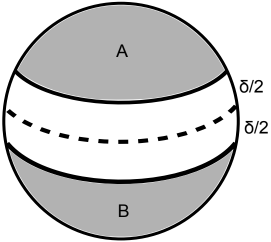

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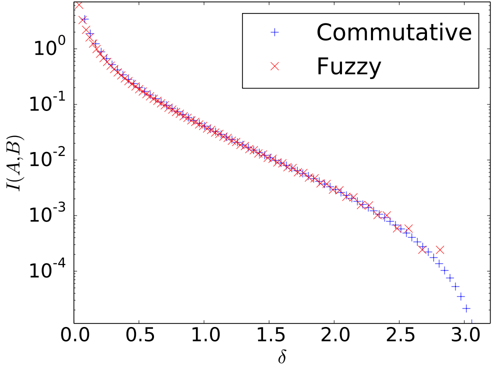

Very *symmetric* situation: probably does not match for off-centre strip.