Entanglement Negativity

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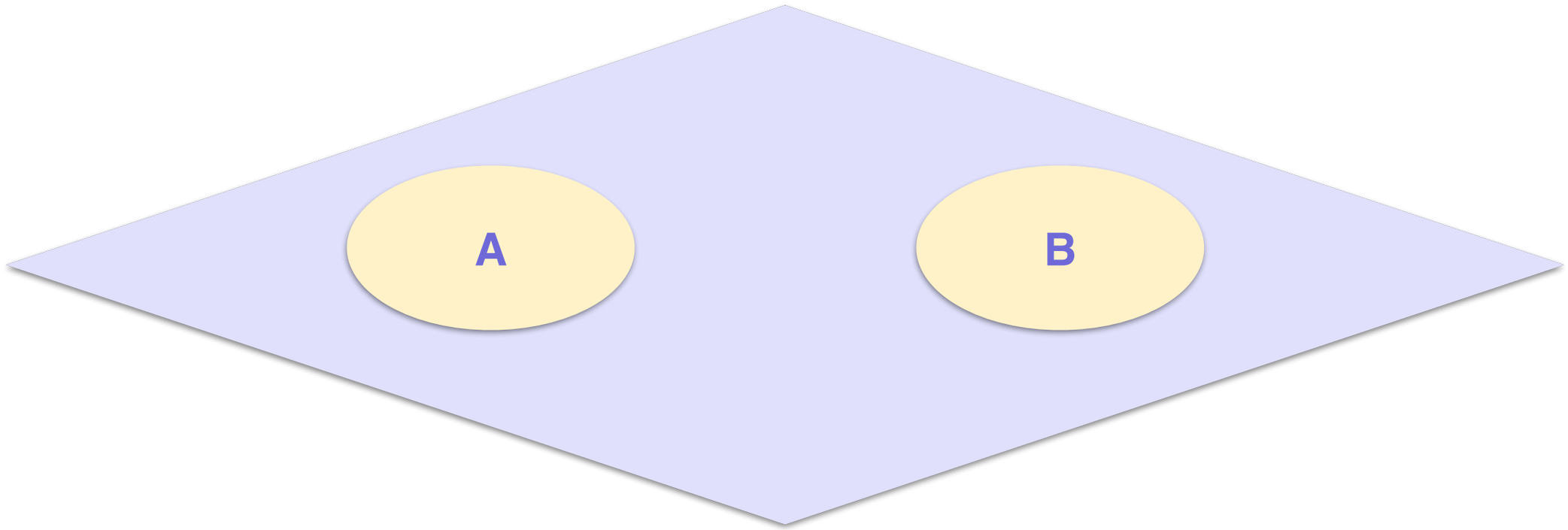
20 August 2014
Quantum Information in Quantum Gravity - Vancouver

Based on 1406.6989 with Mukund Rangamani

Entanglement between subregions

$$S_A \neq S_B$$



Partial transpose of a density matrix:

$$\rho \in \mathcal{H}_A \otimes \mathcal{H}_B \qquad \left\langle i^A l^B \right| \tilde{\rho} \left| j^A m^B \right\rangle = \left\langle i^A m^B \right| \rho \left| j^A l^B \right\rangle$$

PPT criterion:
if a state is separable, the partial transpose has positive eigenvalues

Peres '96

Distillability:
if a state has positive partial transpose, it is undistillable

Horodecki et al. '98

Logarithmic Negativity:

$$\mathcal{E} = log \left\| \tilde{\rho} \right\|$$

Vidal & Werner '02

- Only sensible to distillable entanglement
- Upper bound to the rate of entanglement distillation
- Bigger than entanglement entropy for pure states
- Equal to entanglement entropy for maximally entangled states

Negativity of the eternal BH

$$|TFD\rangle_\beta = \frac{1}{\sqrt{Z(\beta)}} \sum_n e^{-\frac{\beta}{2}E_n} |E_n\rangle_L \otimes |E_i\rangle_R$$

$$\mathcal{E}_\beta = \beta \left(F(\beta) - F(\beta/2) \right)$$

In the high temperature limit the TFD state is maximally entangled and the negativity is equal to the entanglement entropy

Generalization to any bipartite pure state

$$\mathcal{E}(\tilde{\rho}) = S_{1/2}(\rho_R)$$

Calabrese, Cardy, Tonni ‘12

Negativity of a ball region in the vacuum:
calculation for CFTs in various dimensions at strong and weak coupling

Casini & Huerta '10
Casini, Huerta, Myers '11
Hung, Myers, Smolkin, Yale '11
Klebanov, Pufu, Sachdev, Safdi '12
Fursaev '12

Comparing the negativity to the entanglement entropy:

$$\chi_d = \left| \frac{C_{\text{univ}}\left[\mathcal{E}(\tilde{\rho})\right]}{C_{\text{univ}}\left[S(\rho_R)\right]} \right|$$

- Bigger than one in quantum mechanics
- Conjecture: this is true also for the universal terms in QFTs
- Further evidence from Rényi perturbation theory

Lewkowycz & Perlmutter '14

- The ratio decreases at strong coupling
- Meaning of the ratio?