Entanglement Entropy in Nonlocal Theories

Charles Rabideau

Based on work with Joanna Karczmarek
1307.3517 and work in progress

Department of Physics and Astronomy, University of British Columbia

Volume Laws

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- Motivations:
  - Violations of area law
  - Interest in noncommutative theories
  - Scrambling

Theories Considered

hep-th/9907166
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  - Dipole length θp.

- Dipole Theory:

$$(f \tilde{\star} g)(\vec{x}) = f\left(\vec{x} - \frac{\vec{L}_g}{2}\right) g\left(\vec{x} + \frac{\vec{L}_f}{2}\right)$$

  - Fixed dipole length L.

Features of the Gravity Duals

- Non trivial geometry of the compact dimensions and dilaton

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$\Longrightarrow$ Open and closed string metric

Causality is determined by closed string metric

$$\frac{\mathrm{d}s^2}{R^2} = u^2\left(-\mathrm{d}t^2 + \mathrm{d}z^2 + f(u)\left[\mathrm{d}x^2 + \mathrm{d}y^2\right]\right) + \frac{\mathrm{d}u^2}{u^2} + \dots$$

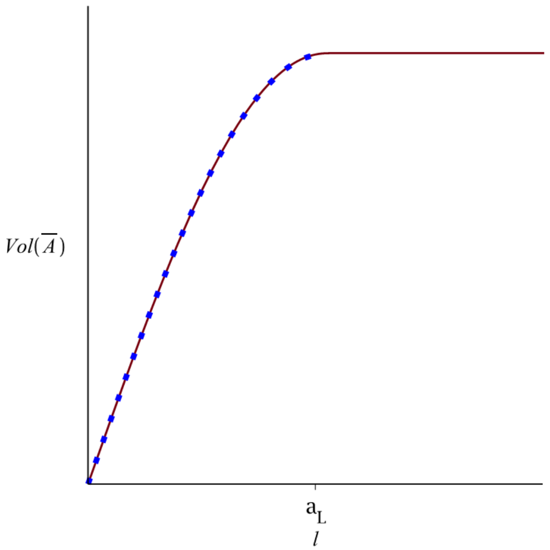
$$f^{-1}(u) = 1 + (a_\theta u)^4 \qquad a_\theta = \lambda^{\frac{1}{4}}\theta^{\frac{1}{2}}$$

$$f^{-1}(u) = 1 + (a_L u)^2 \qquad a_L = \lambda^{\frac{1}{2}}L$$

Entanglement Entropy of a Strip

- Dipole:

$$S_A \begin{array}{l} \xrightarrow{l\to 0} \frac{N^2}{2\pi}\,|A|\epsilon^{-3} \\ \xrightarrow{l\to\infty} \frac{N^2}{3\pi}\,a_L|\partial A|\epsilon^{-3} \end{array}$$



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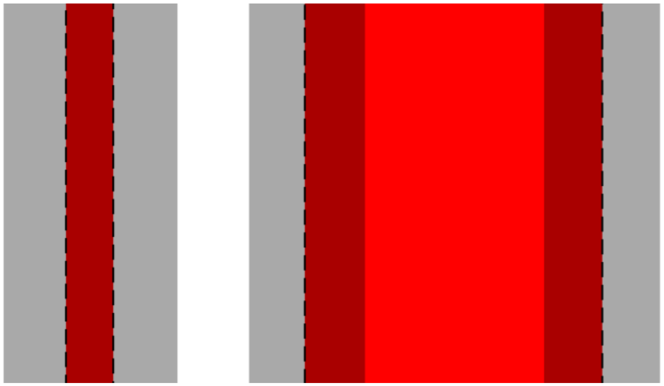

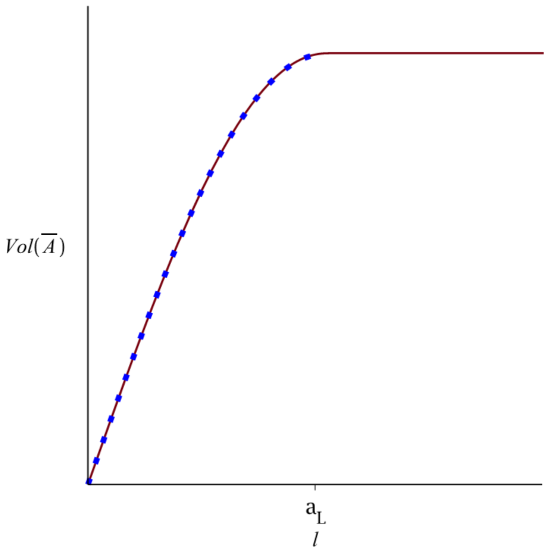

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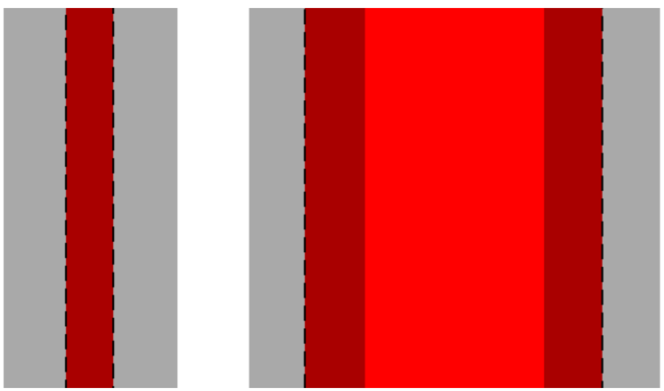

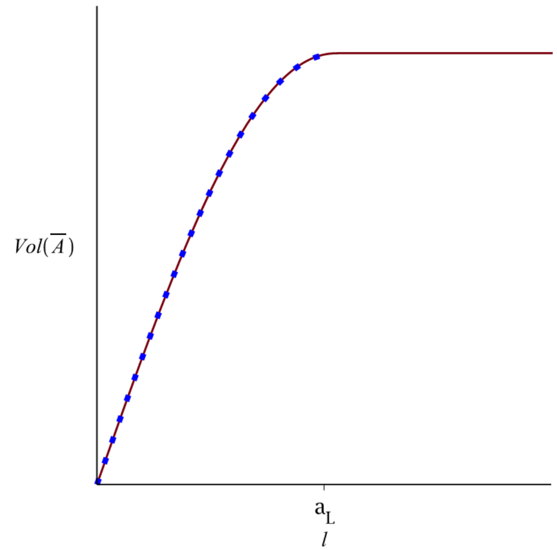

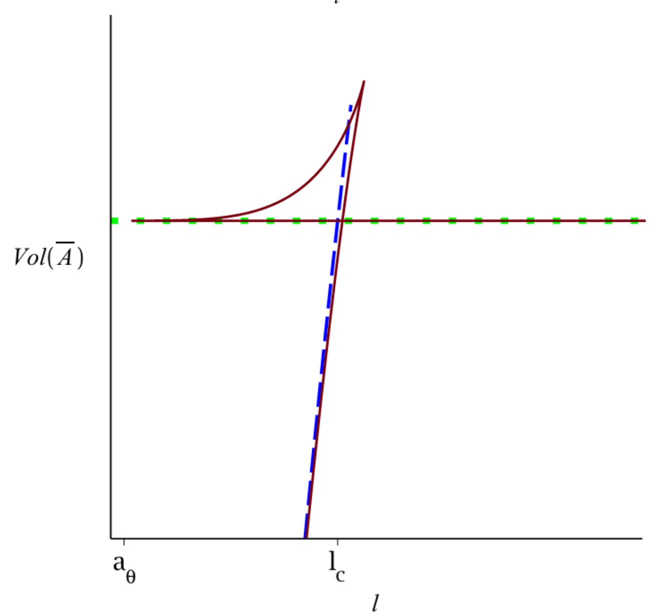

- Noncommutative
  - First order at $l_c = \frac{1}{2}a_\theta^2/\epsilon$.
  - $S_A = \frac{N^2}{2\pi}\frac{|A|}{\epsilon^3}$ for all l as $\epsilon \to 0$.

Field Theory Calculation

$$S = \int \left[\partial\phi \star \partial\phi + \lambda\phi \star \phi \star \phi \star \phi\right]$$

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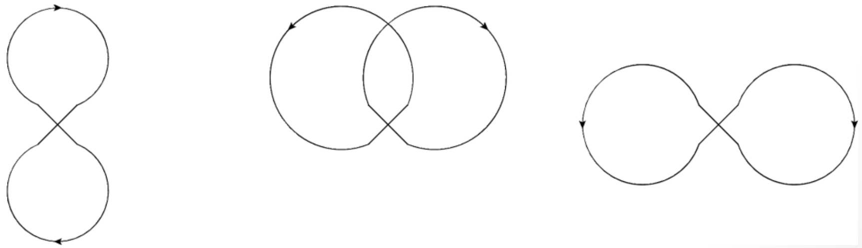
- Does intuitive picture extent to this case? $S_A \propto \frac{l_c|\partial A|}{\epsilon^d}$?
- No! Leading divergence is same order as local theory
- Known story:
  - Only non-planar diagram gives new contribution
  - No higher order UV divergences
  - This story can be extended to gauge fields



The End

- Noncommutative field theory exhibits a volume law in strong coupling, large N limit.
- Intuitive picture for understanding the volume law.

Gravity Duals

- Noncommutative field theory:

$$\frac{ds^2}{R^2} = u^2 \left(-dt^2 + dz^2 + f(u)\left[dx^2 + dy^2\right]\right) + \frac{du^2}{u^2} + d\Omega_5^2$$

$$e^{2\phi} = g_s^2 f(u) \qquad B_{xy} = \frac{1}{\theta}\left(1 - f(u)\right)$$

$$f(u) = \frac{1}{1 + (a_\theta u)^4}$$

- Dipole Theory:

$$\frac{ds^2}{R^2} = u^2 \left(-dt^2 + dy^2 + dz^2 + f(u)dx^2\right) + \frac{du^2}{u^2} + f(u)d\psi^2 + V_{\mathbb{C}\mathbf{P}^2}$$

$$e^{2\phi} = g_s^2 f(u) \qquad B_{x\psi} = -\frac{1}{L}\left(1 - f(u)\right)$$

$$f(u) = \frac{1}{1 + (\lambda L u)^2}, \qquad \int V_{\mathbb{C}\mathbf{P}^2} = \frac{\pi^2}{2}.$$