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Entanglement Entropy of Local operator Excited states in rational CFTs

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motivation

- To study the property of entanglement entropy for excited states . [cf. Calabrese, Cardy 05, 07: Time evolution of excited states called "Quantum Quench".]

→ In this talk, we consider local operator excited states:

$$|O\rangle \equiv O(x)\,|\mathrm{vac}\rangle \quad (t=0)$$

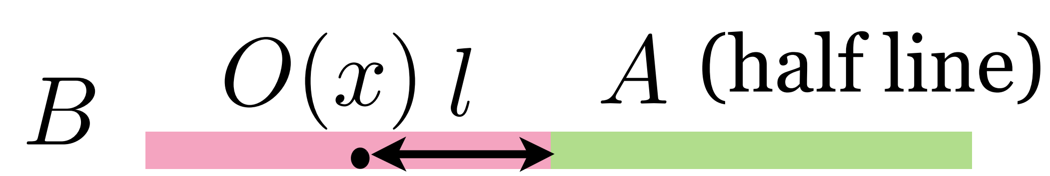
- To study the universal property of entanglement entropy in the limit the subsystem is very large.

cf) In the small size limit , there is a property analogous to the first law of thermodynamics: $\Delta S_A[|O\rangle] \propto E_O$

[Bhattacharya-Nozaki-Takayanagi-Ugajin 12]
[Blanco-Casini-Hung-Myers 13]

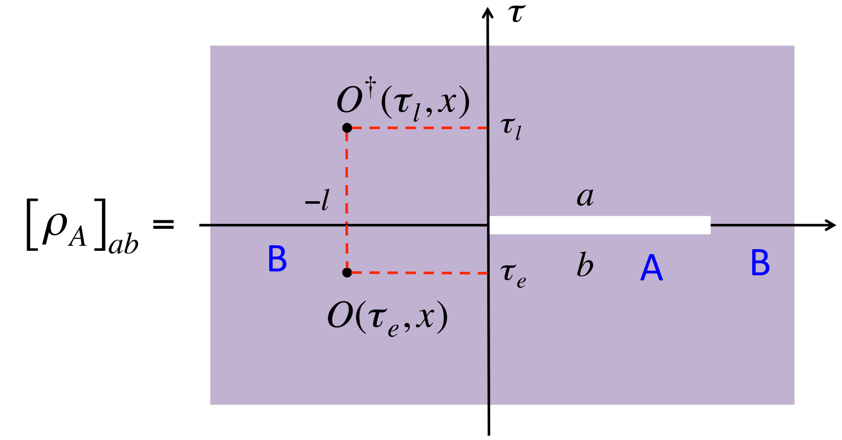
($\Delta S_A[|O\rangle] = S_A[|O\rangle] - S_A[|\mathrm{vac}\rangle]$)

→ In this talk , we consider the following setup:



Introduction

- Replica method for excited states



State: $e^{-\varepsilon H}\cdot e^{-iHt}O(x)\left|0\right\rangle$

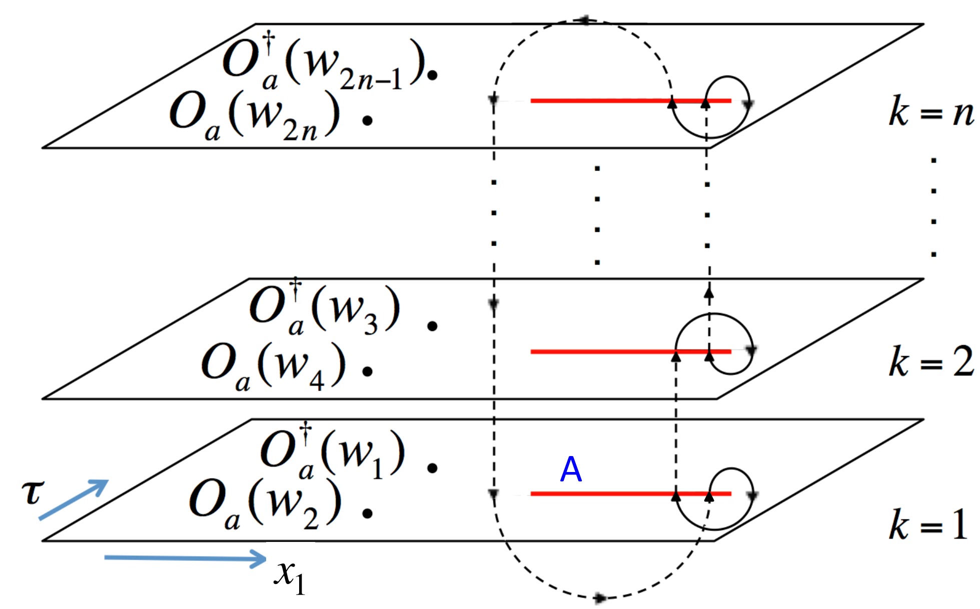
Density matrix: $\rho_{tot}(t,x)=e^{-iHt}e^{-\varepsilon H}O(x)\left|0\right\rangle\left\langle 0\right|O^\dagger(x)e^{-\varepsilon H}e^{iHt}$

$$=O(\tau_e,x)\left|0\right\rangle\left\langle 0\right|O^\dagger(\tau_l,x)\ (\tau_e=-\varepsilon-it,\ \tau_l=\varepsilon-it)$$

Final results

$$\Delta S_A^{(n)}=\frac{1}{1-n}\times$$
$$[\log\left\langle O^\dagger(w_1)O(w_2)\cdots O^\dagger(w_{2n-1})O(w_{2n})\right\rangle_{\Sigma_n}$$
$$-n\log\left\langle O^\dagger(w_1)O(w_2)\right\rangle_{\Sigma_1}$$

[Nozaki-TN-Takayanagi 14]



w_{2k-1}, w_{2k} :the coordinate of the inserted local operator on the k-th sheet .

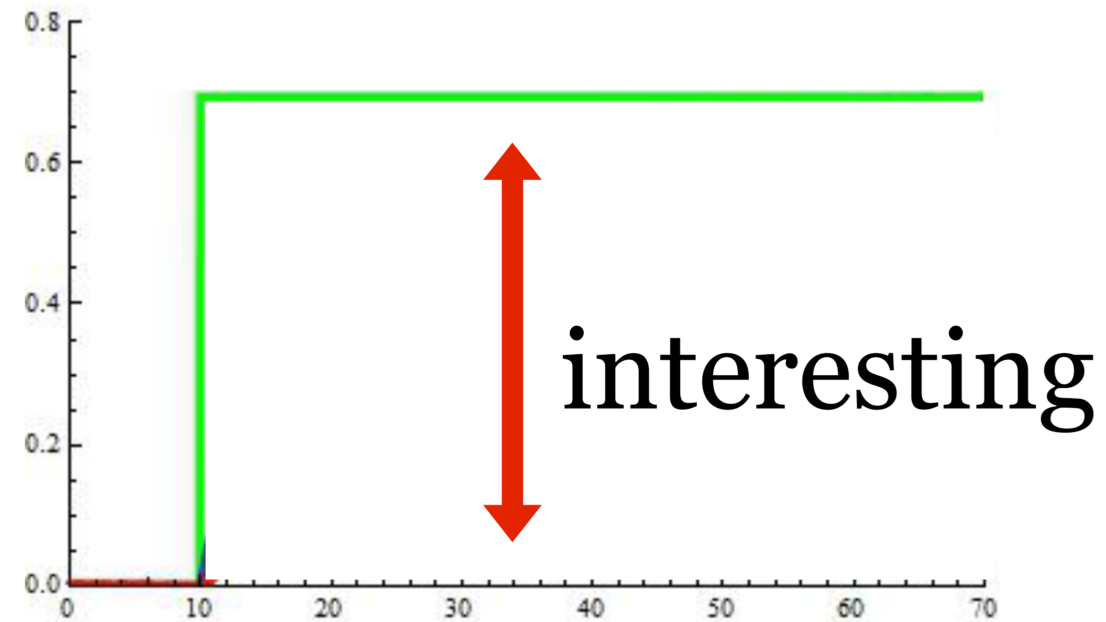
introduction

- 2D free massless scalar: $$S = \frac{1}{2}\int d^2x \partial_\mu \phi \partial^\mu \phi$$

Time evolution of entanglement entropy

operator: $$O(x) = e^{i\alpha\phi} + e^{-i\alpha\phi}$$

$\Delta S_A^{(2)}$



Results

$$\Delta S_A^{(n)} = \begin{cases} 0 & (t < l) \\ \log 2 & (t > l) \end{cases}$$

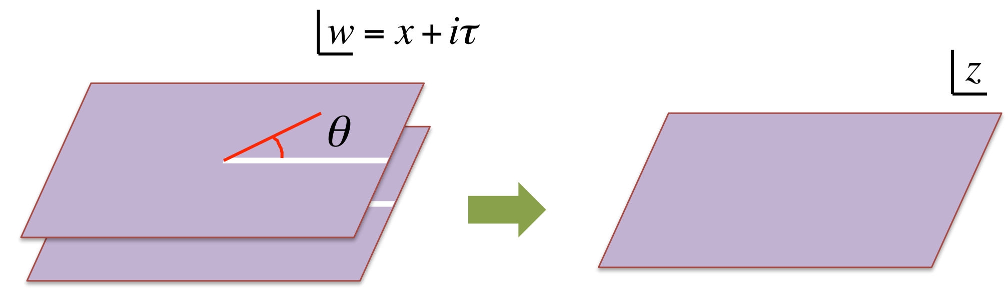
[Nozaki-TN-Takayanagi 14]

universal: shape of function (step function)

theory and operator dependence : **final value**

➡ explicit calculation in Rational CFTs!

Results in 2d RCFTs



Using the conformal mapping

$$\Sigma_2 \to \Sigma_1 : z = \sqrt{w} = \sqrt{re^{i\theta}} (0 \leq \theta < 4\pi)$$

We can write REE in terms of 4 pt function on $\Sigma_1 = \mathbb{C}$!

$$\langle O(w_1, \bar{w}_1) O(w_2, \bar{w}_2) O(w_3, \bar{w}_3) O(w_4, \bar{w}_4) \rangle_{\Sigma_2}$$

$$= |z_{13} z_{24}|^{-4\Delta_O} G_O(z, \bar{z})$$

($z = z_{12} z_{34} / z_{13} z_{24}$:cross ratio)

$G_O(z, \bar{z})$ can be calculated by expanding in terms of conformal block and we get the late time value:

$$\Delta S_A^{(2)} = \log d_O$$

d_a :quantum dimension of O_a

fusion rule: $O_a \times O_b = \sum_c \mathcal{N}_{ab}{}^c O_c$

⟶ number of operators in $\#[O_a]^k \sim (d_a)^k$

⟶ d_a operators in O_a

➡ d_a :number of operators in O_a

~ degrees of freedom of O_a !

$$\boxed{\Delta S_A = \log d_a} \sim \log(\mathrm{d.o.f})$$

Conclusion

- When the subsystem is very large , the late time value of $\Delta S_A^{(n)}$ becomes finite.
- $\Delta S_A^{(n)}$ is the contribution to EE from the local operator,and (R)EE can detect the degrees of freedom of local operator.

cf) EE for ground states can degrees of freedom of theory

(for example central charge)