Quantum Information in Quantum Gravity @ UBC

Quantum Entanglement of Local Operators

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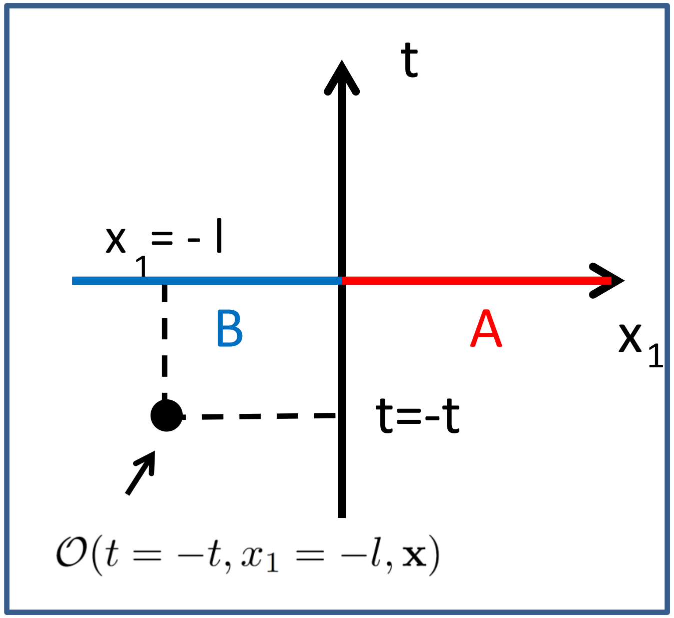
1. Based on arXiv:1401.0539v1 [hep-th] (Phys. Rev. Lett. 112, 111602 (2014)) with Tokiro Numasawa, Tadashi Takayanagi

2. Based on arXiv:1405.5875 [hep-th]

Motivation

We study the property of

Renyi Entanglement Entropy (R)EE.



1. The size of subsystem is ***infinite***.

A half of the total system:

$$x^1 \geq 0$$

2. A state is defined by acting a local operator on the ground state:

$$|\Psi\rangle = \mathcal{N}^{-1}\mathcal{O}(t=-t, x_1=-l, \mathbf{x})\,|0\rangle\,.$$

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We study the property of

Renyi Entanglement Entropy

(R)EE

1

$$\Delta S_A^{(n)} \Longrightarrow \textit{Some Constants}$$

At late time

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on the ground state:

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Motivation

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(R)EE

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$$\Delta S_A^{(n)} \Rightarrow$$

At late time

Some Constants

Unique Behavior

2.

$$|\Psi\rangle = \mathcal{N}^{-1}\mathcal{O}(t=-t, x_1=-l, \mathbf{x})\,|0\rangle\,.$$

The definition of $\Delta S_A^{(n)}$

$\Delta S_A^{(n)}$ is defined by the excess of REE:

$$\Delta S_A^{(n)} = S_A^{(n)Ex} - S_A^{(n)G},$$

where

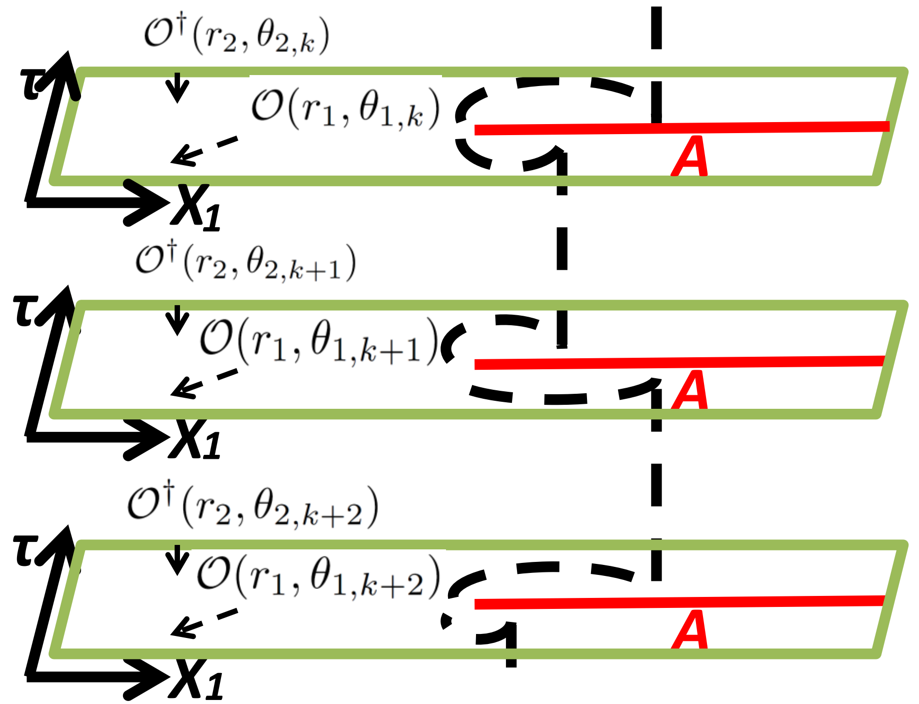
- REE for $|\Psi\rangle = \mathcal{N}^{-1}\mathcal{O}(t, x^1)\,|0\rangle$:

$$S_A^{(n)Ex} \sim \frac{1}{1-n}\log\left[\frac{\int D\Phi\mathcal{O}^\dagger(r_1,\theta_{1,1})\mathcal{O}(r_2,\theta_{2,1})\cdots\mathcal{O}^\dagger(r_1,\theta_{1,n})\mathcal{O}(r_2,\theta_{2,n})}{\left(\int D\Phi\mathcal{O}^\dagger(r_1,\theta_{1,1})\mathcal{O}(r_2,\theta_{2,1})\right)^n}\right]$$

- REE for Ground State:

$$S_A^{(n)} \sim \frac{1}{1-n}\log\left[\frac{Z_n}{Z_1^n}\right]$$

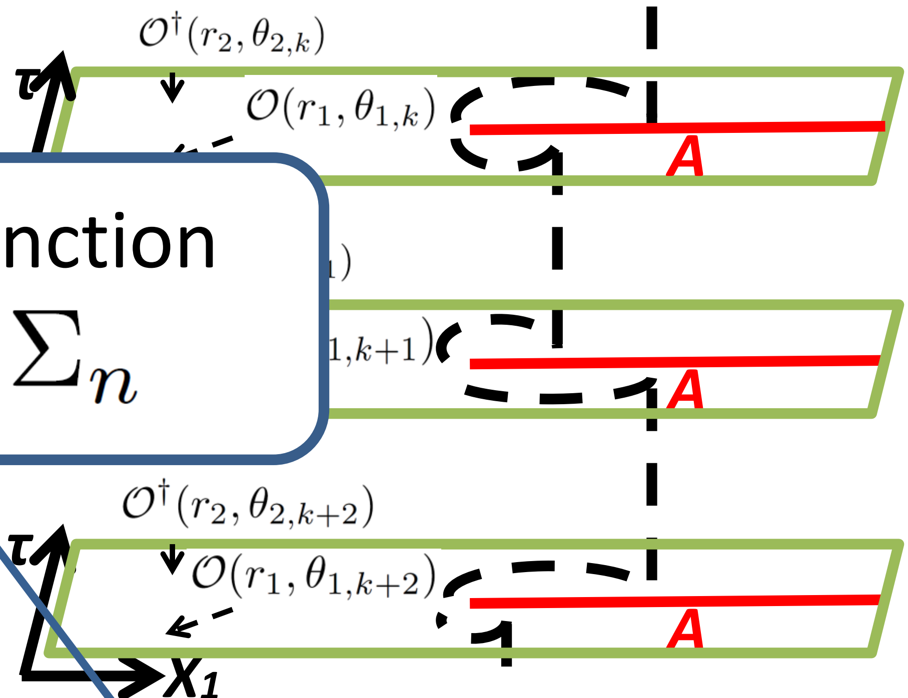
The definition of $\Delta S_A^{(n)}$



$$\theta_{1,i} = \theta_1 + 2\pi(i-1)$$
$$\theta_{2,i} = \theta_2 + 2\pi(i-1)$$

$$\Delta S_A^{(n)} = \frac{1}{1-n}\left(\log \left\langle \mathcal{O}^\dagger(r_2,\theta_{2,n})\mathcal{O}(r_1,\theta_{1,n})\cdots\mathcal{O}^\dagger(r_2,\theta_{2,1})\mathcal{O}(r_1,\theta_{1,1})\right\rangle_{\Sigma_n} - n\log\left\langle \mathcal{O}^\dagger(r_2,\theta_2)\mathcal{O}(r_1,\theta_1)\right\rangle_{\Sigma_1}\right).$$

The definition of $\Delta S_A^{(n)}$



2n-Point Function on Σ_n

$$\Delta S_A^{(n)} =$$

$$\theta_{1,i} = \theta_1 + 2\pi(i-1)$$
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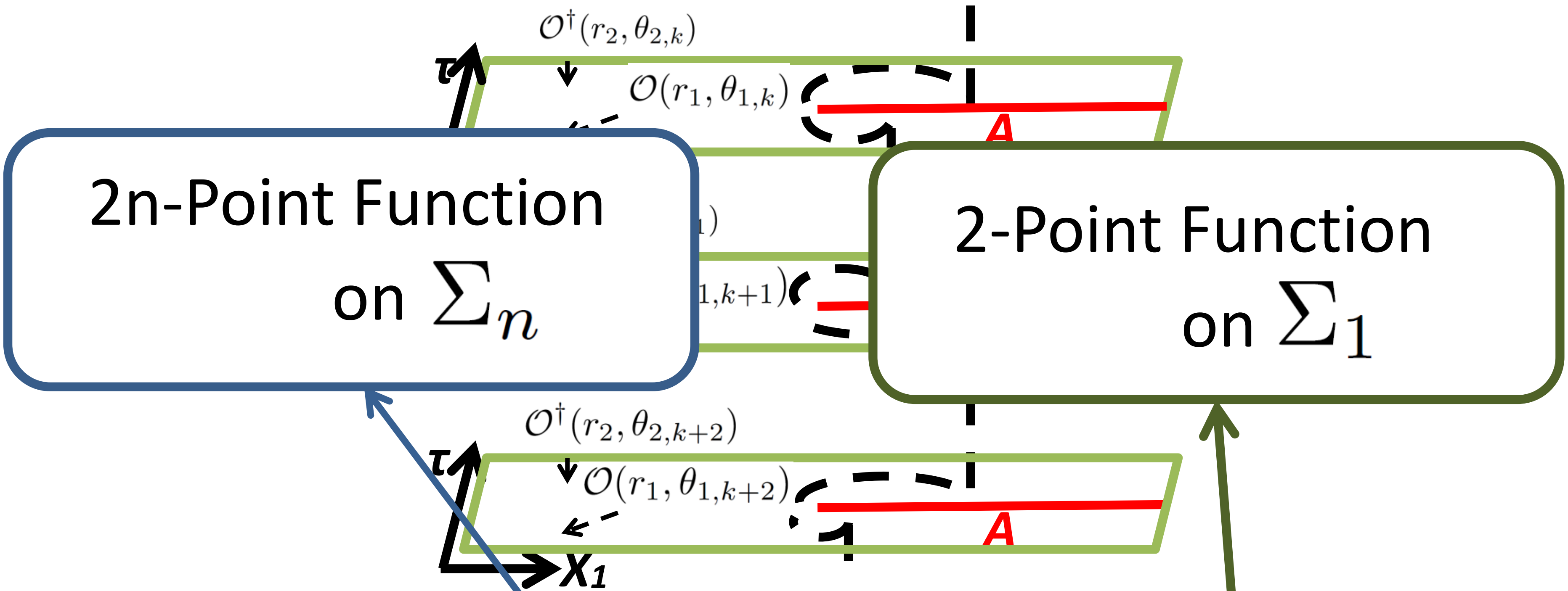
$$\left. - n\log\left\langle \mathcal{O}^\dagger(r_2,\theta_2)\mathcal{O}(r_1,\theta_1)\right\rangle_{\Sigma_1}\right).$$

The definition of $\Delta S_A^{(n)}$



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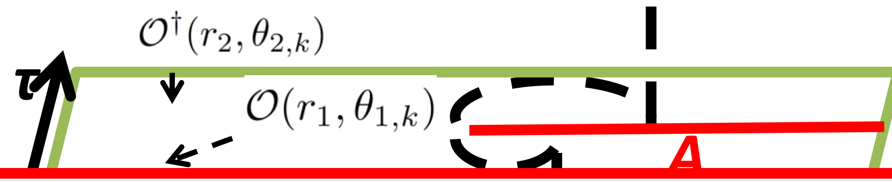
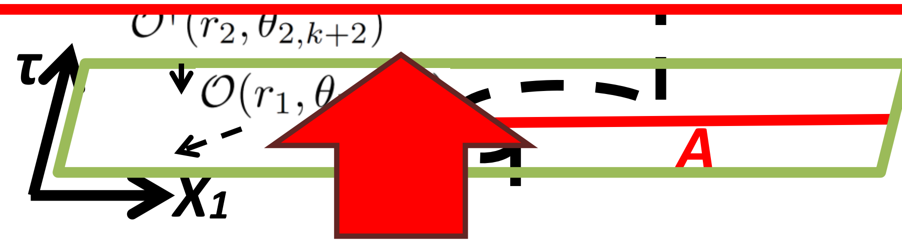
Replica Method



This formula holds for any local operators in general QFT in any dimensions.



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Example

We consider ***free massless scalar*** field theory in ***d+1 dim.***

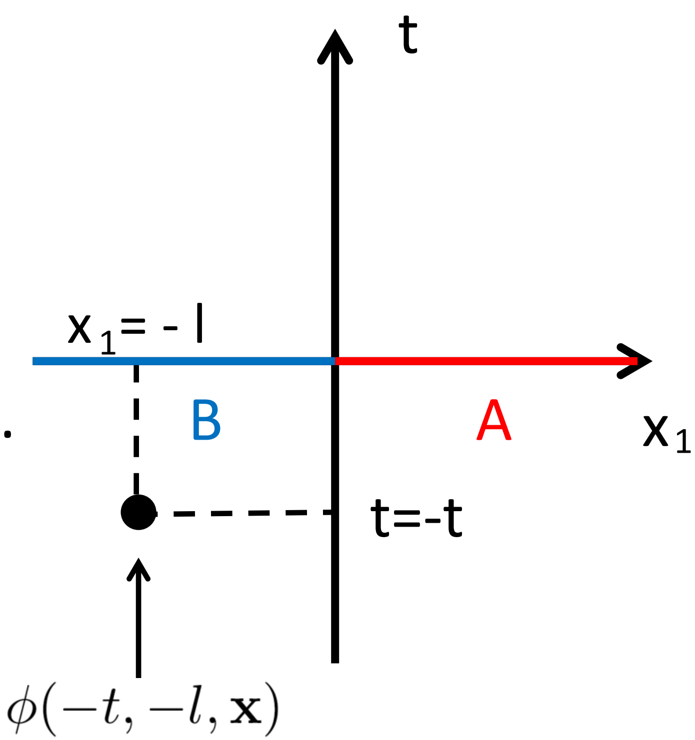
Especially, we focus on that in ***4 dim.***

We act a local operator $\phi(-t,-l,\mathbf{x})$ on the ground state:

$$|\Psi\rangle = \mathcal{N}^{-1}\phi(-t,-l,\mathbf{x})\,|0\rangle\ .$$

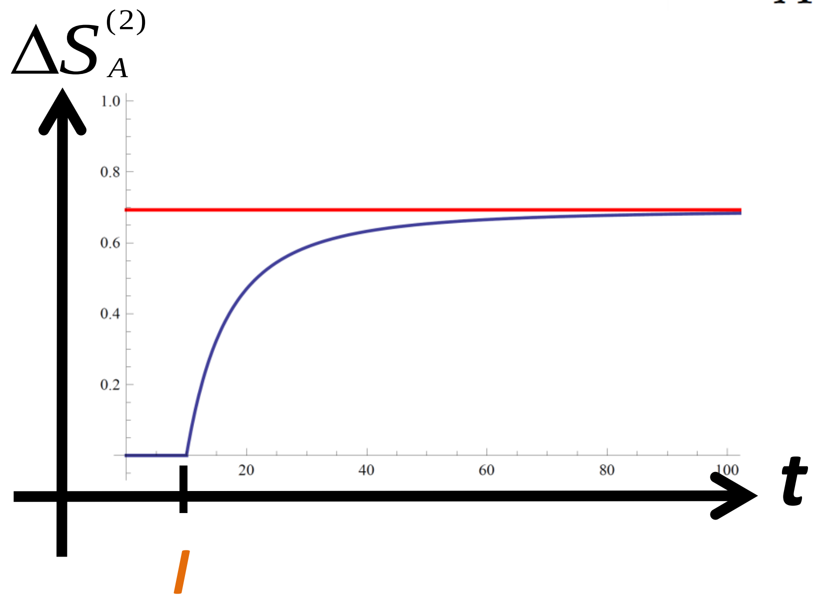
Subsystem A : $x^1 \geq 0$

We measure the second (Renyi) entanglement entropies $\Delta S_A^{(2)}$ at t=0.

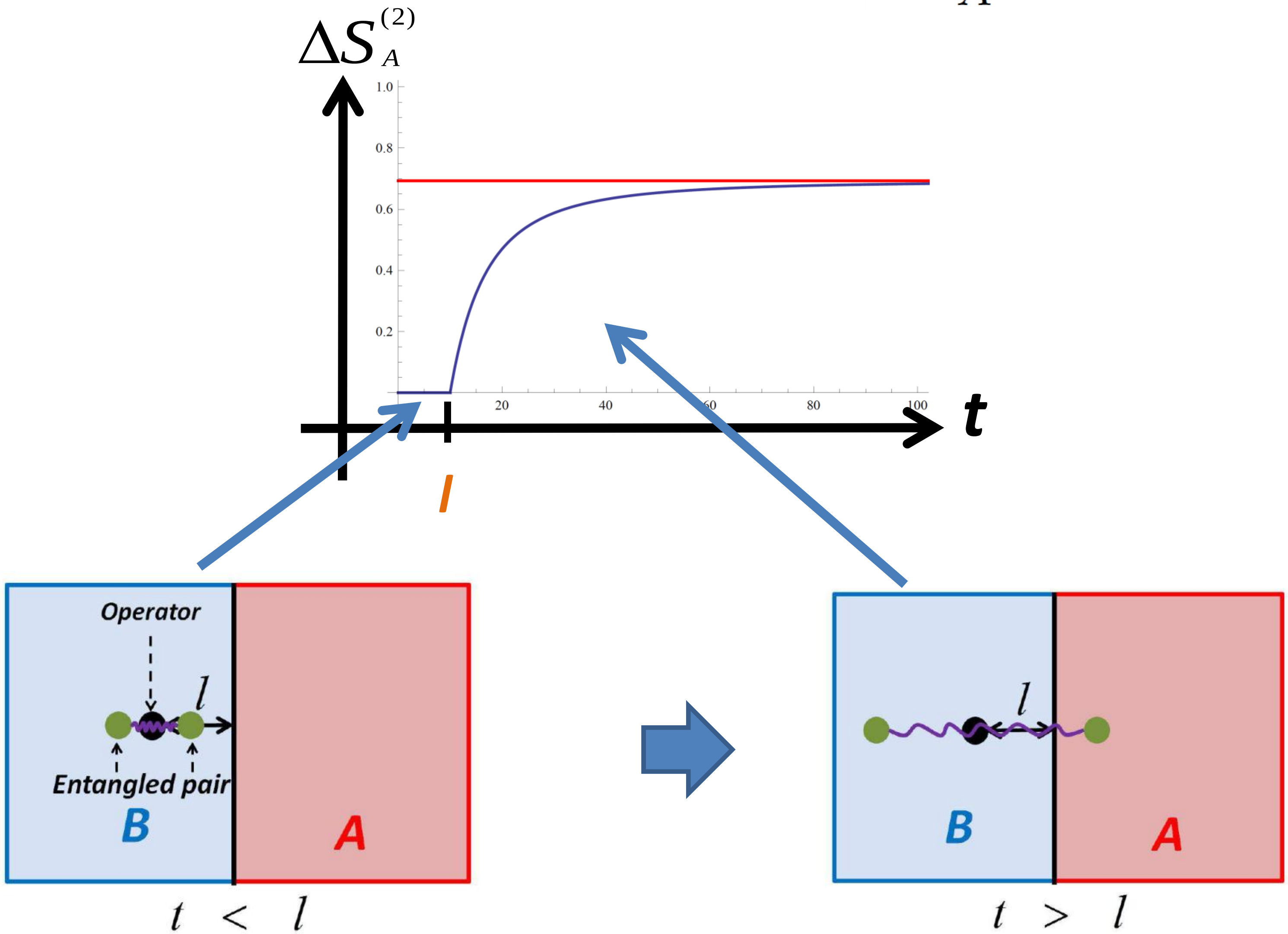


⇨ ***Time evolution!!***

Time Evolution of $\Delta S_A^{(2)}$

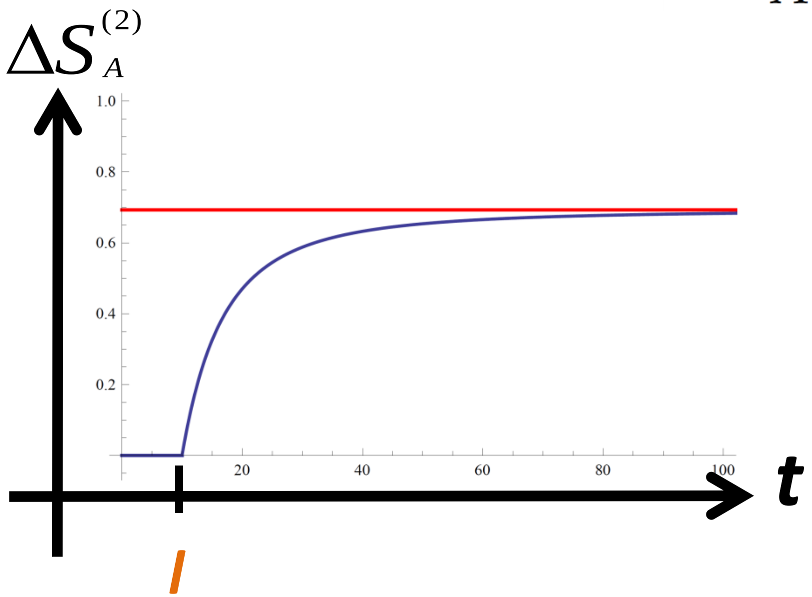


Time Evolution of $\Delta S_A^{(2)}$



We can interpret this behavior in terms of ***the relativistic propagation of an entangled pair***.

Time Evolution of $\Delta S_A^{(2)}$

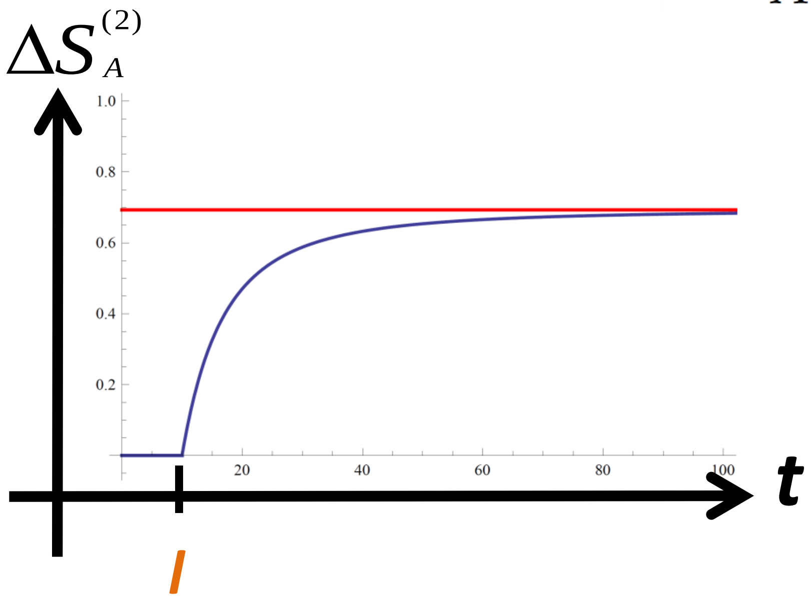


$\Delta S_A^{(2)}$ for $|\Psi\rangle = \mathcal{N}^{-1}\phi(-t,-l,\mathbf{x})\,|0\rangle$ approaches ***constants! !***

(log2)

We call them ***the (Renyi) entanglement entropies of operators*****.**

Time Evolution of $\Delta S_A^{(2)}$



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Generalize Results

We defined ***the (Renyi) entanglement entropies of operators*** by the late time values of $\Delta S_A^{(n)}$.

The (Renyi) entanglement entropies of specific operators ($:(\partial^m \phi)^k:$) which are composed of single species operator are given by

$$\boxed{\begin{aligned} \Delta S_A^{(n)f} &= \frac{1}{1-n} \log \left(\frac{1}{2^{nk}} \sum_{j=0}^{k} \left({}_k C_j \right)^n \right). \\ \Delta S_A &= k \cdot \log 2 - \frac{1}{2^k} \sum_{j=0}^{k} {}_k C_j \log {}_k C_j . \end{aligned}}$$

for ***any dimension.***

➡ They characterize the local operators from the viewpoint of quantum entanglement!!

Sum rule

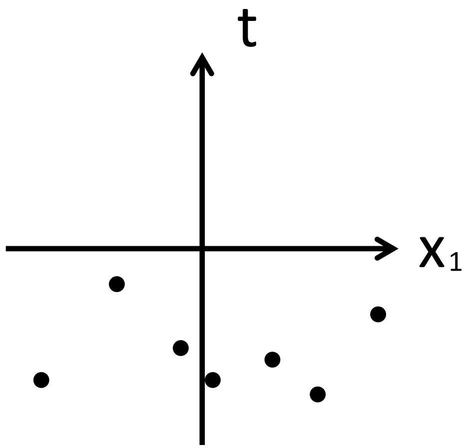
We acts various local operators on the ground state.

A locally excited state is given by $|\psi\rangle = \mathcal{N}^{-1}\mathcal{T}\prod_{i}^{k}\mathcal{O}^i(t^i, x^{1,i})\,|0\rangle$.

Time ordering Op.

The late time of $\Delta S_A^{(n)}$ is given by

$$\boxed{\Delta S_A^{(n)} = \sum_{i=1}^{k}\Delta S_A^{(n)i}}$$



Local operators

They are given by the sum of the REE for the state defined by acting each operators $\mathcal{O}^i(t^1, x^{1,i})$ on the ground state.

Summary

- We defined the (Renyi) entanglement entropies of local operators.

  -They characterize local operators from the viewpoint of quantum entanglement.

- These entropies of the operators (constructed of single-species operator) is given by the those of binomial distribution.

  -The results we obtain in terms of entangled pair agree with the results we obtain by replica method.

- They obey the sum rule.

Future Problems

- The formula for the operators constructed of multi-species operators: $:(\partial_r^m \phi)^k \phi^l:$ (generally depend on the spacetime dimension).

- The (Renyi) entanglement entropies of operators in the interacting field theory . (also massive and finite temp.)

- The (Renyi) entanglement entropies for the excited state defined by acting non-local operators.