Spectra of orbifold CFTs

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UBC Vancouver | 20th August 2014

work in progress with M. Rangamani and E. Martinec

- Holographic CFTs have universal features
  - e.g. [Hartman '13]: $c \to \infty$ and # light states small

$$\Downarrow$$

EE and ERE universal

- Nice class of such theories (e.g. D1-D5 system):
  symmetric product orbifold of 2d CFTs
  - Broad goal: for orbifold CFTs understand these statements in much greater detail

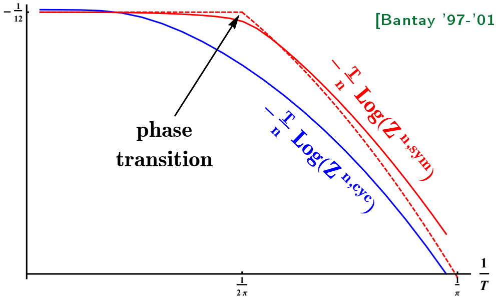
Orbifold CFTs on torus $\mathbb{T}_\tau$

cf. [Keller '11]

[Dijkgraaf-Moore-Verlinde^2 '96]

[Bantay '97-'01]

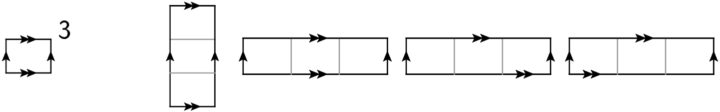
CFT	partition function	central charge
$\mathcal{C}$	$Z(\tau)$	c
$\mathcal{C}^{\otimes n}$	$Z(\tau)^n$	nc
$\mathcal{C}^{\otimes n}/S_n$	$Z^{n,sym}(\tau)$	nc
$\mathcal{C}^{\otimes n}/\mathbb{Z}_n$	$Z^{n,cyc}(\tau)$	nc

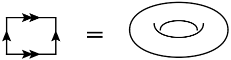


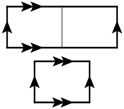
$$Z^{n,sym/cyc}(\tau) \simeq \sum_{\substack{\hat{\mathbb{T}}:\ \text{unbranched } n\text{-sheeted} \\ \text{covers of } \mathbb{T}_\tau,\ \text{deter-} \\ \text{mined by } S_n, \mathbb{Z}_n}} \left(c(\hat{\mathbb{T}}) \prod_{\substack{\hat{\mathbb{T}}_{\hat{\tau}}:\ \text{connected} \\ \text{components of } \hat{\mathbb{T}}}} Z(\hat{\tau}) \right)$$

- E.g. $\mathbb{Z}_3$-orbifold:

$$Z^{3,cyc}(\tau) = \frac{1}{3} Z(\tau)^3 \; + \; \frac{2}{3} \left[Z(3\tau) \; + \; Z\left(\frac{\tau}{3}\right) \; + \; Z\left(\frac{\tau+1}{3}\right) \; + \; Z\left(\frac{\tau+2}{3}\right) \right]$$

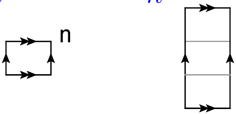


where 

- C.f. S_3-orbifold where more covers exist:  ...

- At **large n**, these formulae enormously simplify:
  - $\mathbb{Z}_n$-orbifold theories:

$$Z^{n,cyc}(\tau) \approx \frac{1}{n} Z(\tau)^n + \frac{n-1}{n}\, e^{-cn\beta/12}$$



$\rightarrow$ degeneracies of low lying states grow with n

  - S_n-orbifold theories:

$$e^{cn\beta/12}\ Z^{n,sym}(\tau) \approx e^{c(n+1)\beta/12}\ Z^{n+1,sym}(\tau)$$

$\rightarrow$ low-lying spectrum becomes n-independent

- This explains why for $n \rightarrow \infty$ the spectrum of $\mathcal{C}^{\otimes n}/S_n$ is universal, but that of $\mathcal{C}^{\otimes n}/\mathbb{Z}_n$ isn't.

- How about orbifold CFTs on surfaces Σ_g with **genus** $\mathbf{g > 1}$? $\left(\begin{smallmatrix}\to \text{ e.g. replica} \\ \text{surfaces!}\end{smallmatrix}\right)$
- The generating formulae basically still hold!

$$Z_g^{n,sym/cyc}(\Sigma_g) \simeq \sum_{\substack{\hat{\Sigma}:\text{ unbranched } n\text{-sheeted} \\ \text{covers of } \Sigma_g,\text{ deter-} \\ \text{mined by } S_n, \mathbb{Z}_n}} \left(c(\hat{\Sigma}) \prod_{\substack{\hat{\Sigma}_{\hat{g}}:\text{ connected} \\ \text{components of } \hat{\Sigma}}} Z_{\hat{g}}(\hat{\Sigma}_{\hat{g}}) \right)$$

$(\hat{g} \in \{g, \ldots, n(g-1)+1\})$

- Idea: **at large** $\mathbf{n}$**, untwisted geometries should dominate again!**
  - Derive large-n asymptotics of $Z_g^{n,sym/cyc}(\Sigma_g)$
  - Similar universal features as for $g = 1$?
  - Gives Rényi entropies of orbifold CFTs
  - Bulk construction: fill in dominating geometries? c.f. [Faulkner '13]