Charged Rényi Entropies

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Based on hep-th/1310.4180 with J. Hung, A. Maloney, R. Myers, S. Matsuura, T. Sierens

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McGill

How does entanglement depend on:

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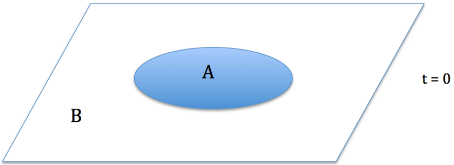
How does entanglement depend on charge distribution?

$\to$ Charged Rényi entropies

Setup

CFT_d with a global $U(1)$ conserved charge.

Consider $|\Psi\rangle = |0\rangle$



Conf. transf. to $S^1 \times \mathbb{H}_{d-1}$

$$\rho_A = e^{-\beta_0 H_E} \quad \Longrightarrow \quad S_{EE} = S_{TH}$$

$$\beta_0 = 2\pi$$

Thermodynamics $\rightarrow$ GRAND CANONICAL ENSEMBLE

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The charged Rényi entropies are defined as

$$\boxed{S_n(\mu) = \frac{1}{1-n} \log \mathrm{Tr} \left[\rho_A \frac{e^{\mu Q_A}}{n_A(\mu)} \right]^n}$$

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Wilson Line along the S^1:

$$B_\tau = -i\mu/2\pi \quad \longrightarrow \int_{S^1} B = -in\mu$$

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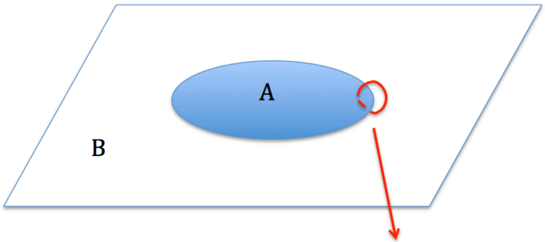

Near ∂A, $B \sim -i\frac{\mu}{2\pi}d\theta$ $\qquad$ $\oint B = -in\mu$

$\longrightarrow$ The twist operator carries 'magnetic flux' along ∂A

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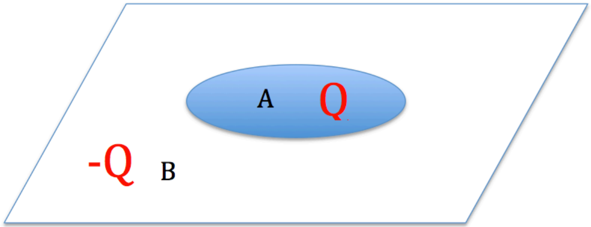

Note that $Q_{tot} = Q_A + Q_B = 0$

Holographic Calculations

$$S_n(\mu) = \frac{n}{n-1}\frac{1}{T_0}\int_{T_0/n}^{T_0} S(T,\mu)dT$$

AdS/CFT $\longrightarrow$ Areas of charged hyperbolic black holes

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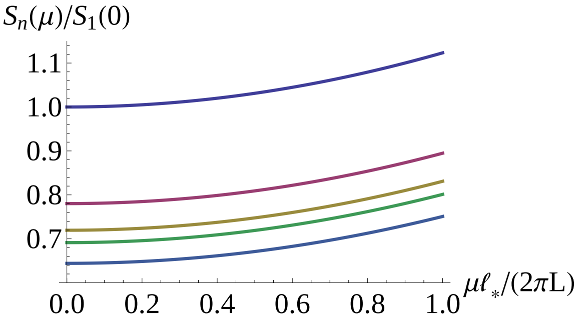

Figure: Top to Bottom: n=1,2,3,4,10

Conclusion and future challenges

- We have found a new measure of entanglement that tracks the charge distribution between subsystems A and B.
- It would be interesting to investigate different states $|\psi\rangle \neq |0\rangle$
- It would also be interesting to try to understand the Wilson Loop as giving a new state, is there a generalized RT surface?

Thank you!