Problem Set 11

Problem 1

Consider a theory with two scalar fields, describing particles of mass m and $M > 2m$ (they might be two kinds of mesons, for example). The Lagrangian density is

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2}m^2\phi^2 + \frac{1}{2}(\partial_\mu \Phi)^2 - \frac{1}{2}M^2\Phi^2 - \mu\phi^2\Phi \ .$$

In this theory, the interaction allows the Φ particles to decay into two ϕ particles.

a) Calculate the lifetime ($1/\Gamma$) of the Φ particle to leading order in perturbation theory. *Don't worry that the potential here is not bounded from below. There may be higher order terms in the potential that don't affect the low energy physics but which make the potential bounded.*

Hint: We have to be a little careful to take into account the fact that the outgoing particles are not distinguishable. When we are integrating over the directions for the outgoing particle momenta, we should then keep in mind that $\vec{q}_i$ and $-\vec{q}_i$ represent the same final state, so we shouldn't count both of these.

b) Can the Φ decay into three ϕ particles if $M > 3m$?

Problem 2

a) For the theory in problem 1, calculate the differential scattering cross section $d\sigma/d\theta$ (where θ is the scattering angle) and total cross section for scattering of scalar ϕ particles in the theory described in question 1, assuming that the initial particles have momenta $\vec{p}$, $-\vec{p}$.

b) Plot the total cross section as a function of center of mass energy (in units of m) assuming that $M = 4m$.

c) What is the leading behavior of $d\sigma/d\theta$ in the limit where $m, E \ll M$?

Hints:

- *We can assume that the final momenta are different from the initial momenta, so we don't need diagrams with contractions between initial and final particles.*
- *Unlike the problem from last week's homework, the initial and final particles in the amplitude have specific momenta associated with them, so switching how the two initial particles are contracted can result in a different contribution*
- *The hint from problem 1 applies here too.*
- *When computing the integral for the total cross section, you will probably want to use Maple/Mathematica.*

- *Up to an overall constant, the total cross section for part b is just a function of E/m, so you should be able to use Mathematica/Maple to plot this function too.*