Problem Set 10

Problem 1

A single harmonic oscillator (of frequency ω) can be thought of as a field theory in 0+1 dimensions. Calculate the Feynman propagator

$$D_F(t_1 - t_2) \equiv \langle 0|T\{x(t_1)x(t_2)\}|0\rangle$$

for this theory.

Problem 2

a) Suppose the harmonic oscillator is initially in the state $a^\dagger|0\rangle$. If we add an cubic interaction term at time $t = 0$ so that the new Hamiltonian is

$$H = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 x^2 + \frac{1}{3!}\lambda x^3 \ ,$$

calculate the transition probability amplitude (to order λ^2) for the system to be found in the state $\frac{1}{\sqrt{3!}}(a^\dagger)^3|0\rangle$ at time $t = T$. Use Wick's theorem and the general expression we derived in class for transition amplitudes. *Hint: many of the ways of pairing up fields give the same result. An efficient way to figure out how many pairings of each type there are is to use the diagrammatic approach I mentioned in class (see also posted notes).*

Problem 3

For scalar field theory with an interaction term $H_I = \int d^3x \lambda \phi^4$, the transition amplitude from state

$$|\psi_i\rangle = a^\dagger_{p_1} a^\dagger_{p_2}|0\rangle$$

at $t = -\infty$ to state

$$|\psi_f\rangle = a^\dagger_{q_1} a^\dagger_{q_2}|0\rangle$$

at $t = \infty$ is given to first order in λ by

$$-i\lambda \int_{-\infty}^{\infty} dt \int d^3x \langle 0|a_{q_1} a_{q_2} \phi(x,t)\phi(x,t)\phi(x,t)\phi(x,t) a^\dagger_{p_1} a^\dagger_{p_2}|0\rangle \ .$$

Calculate this amplitude, showing that the amplitude is zero unless energy and momentum are conserved. Assume that q_i and p_i are all distinct, so that you can ignore contractions between the as and $a^\dagger$s.