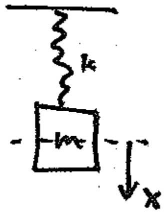
THE HARMONIC OSCILLATOR



Classical physics $\ddot{x} = -\omega x$ $\qquad \omega = \sqrt{\frac{k}{m}}$

$$x(t) = A\cos(\omega t + \phi)$$

$$\text{Energy} = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}m\omega^2 x^2$$

$$= \frac{1}{2}m\omega^2 A^2 \quad \rightarrow \text{can take any value}$$

Quantum mechanics $\quad H = \dfrac{p^2}{2m} + \dfrac{1}{2}m\omega^2 x^2$

$$[x, p] = i\hbar$$

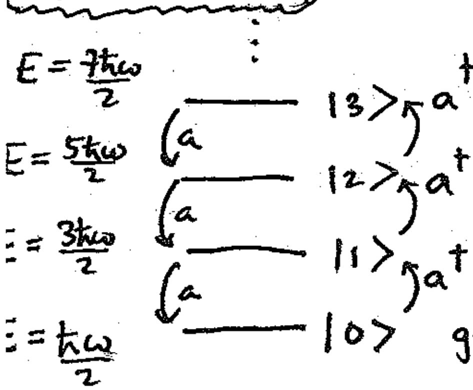
Define $a = \sqrt{\frac{1}{2\hbar m\omega}}(m\omega x + ip)$ ANNIHILATION OPERATOR

$a^\dagger = \sqrt{\frac{1}{2\hbar m\omega}}(m\omega x - ip)$ CREATION OPERATOR

Then $[a, a^\dagger] = 1$ $\qquad [H, a] = -\hbar\omega a$

$[H, a^\dagger] = \hbar\omega a^\dagger$

$$E_n = \hbar\omega\left(n + \tfrac{1}{2}\right)$$



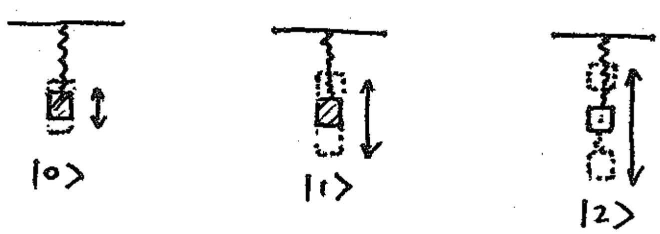
ground state $\quad a|0\rangle = 0$

a and $a^\dagger$ move us between energy eigenstates

$$a^\dagger|n\rangle = \sqrt{n+1}\,|n+1\rangle$$

$$a|n\rangle = \sqrt{n}\,|n-1\rangle$$

$$H|n\rangle = \hbar\omega\left(n + \tfrac{1}{2}\right)|n\rangle$$



Energies and amplitudes QUANTIZED.