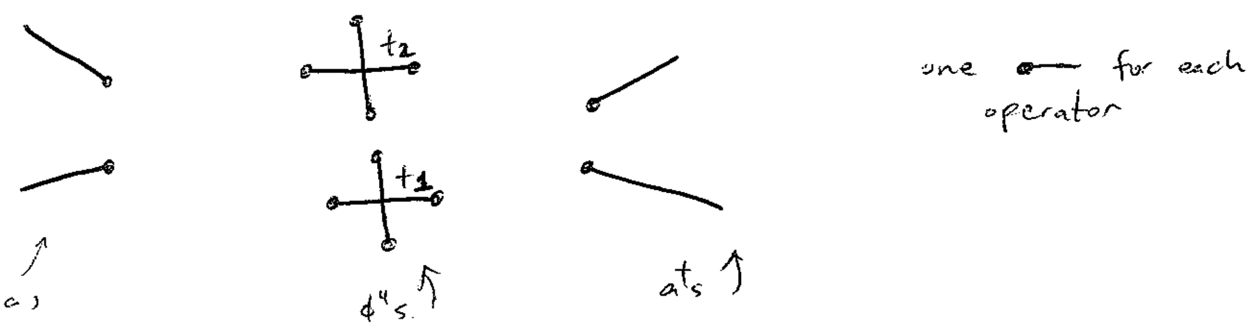
Diagrams for amplitudes:

Consider a term in a transition ampl. such as

$$\langle 0 | a a T\left\{ \int dt_1\, \phi(t_1)\phi(t_1)\phi(t_1)\phi(t_1) \int dt_2\, \phi(t_2)\phi(t_2)\phi(t_2)\phi(t_2) \right\} a^\dagger a^\dagger | 0 \rangle$$

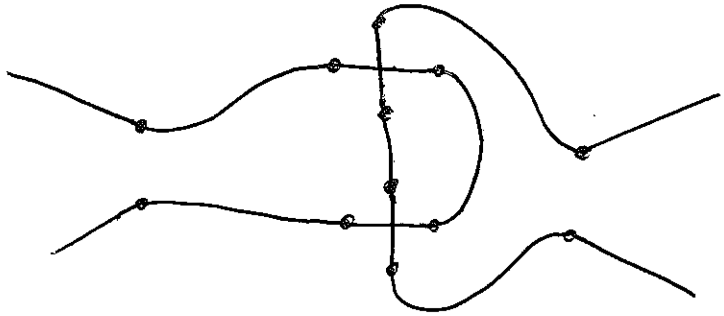
To compute, we sum over all ways of pairing up the operators, and each pairing results in a product of two-point functions. However, for $\langle 0 | A_1 \ldots A_{2n} | 0 \rangle$ we have $(2n-1)(2n-3)\ldots 1 = \frac{(2n)!}{n!\, 2^n}$ ways of making the pairings. In the example above, this is 10395 terms. Fortunately, many of the terms give the same result, so the right way to do the calculation is to figure out how many distinct types of terms we can have (i.e. pairings that lead to distinct results), and then calculate how many pairings lead to this result. This is most easily accomplished using diagrams. For the amplitude above, we write:



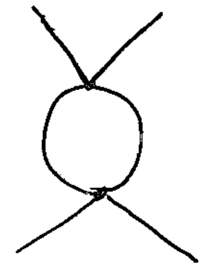
A pairing

$$\langle 0 | a\, a\, \phi(t_1)\phi(t_1)\phi(t_1)\phi(t_1)\ \phi(t_2)\phi(t_2)\phi(t_2)\phi(t_2)\ a^\dagger a^\dagger | 0 \rangle$$

is represented by connecting the corresponding lines on the diagram:



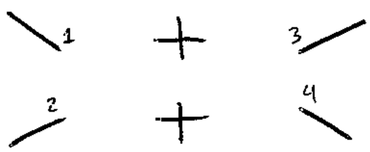
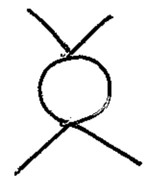
OR



Just from the diagram, we can see that the answer for this pairing is

$$\int dt_1 \int dt_2 \langle a\,\phi(t_1)\rangle \langle a\,\phi(t_2)\rangle \langle T\{\phi(t_1)\phi(t_2)\}\rangle \langle T\{\phi(t_1)\phi(t_2)\}\rangle \langle \phi(t_1)a^{\dagger}\rangle \langle \phi(t_2)a^{\dagger}\rangle$$

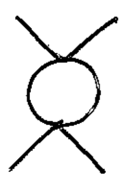
But this same answer comes from any pairing which results in the same diagram. To find the number of such contributions, we need to figure out how many ways we can connect up the lines in

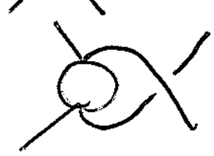
this:  to get this 

The number is $8 \cdot 4 \cdot 6 \cdot 3 \cdot 2$

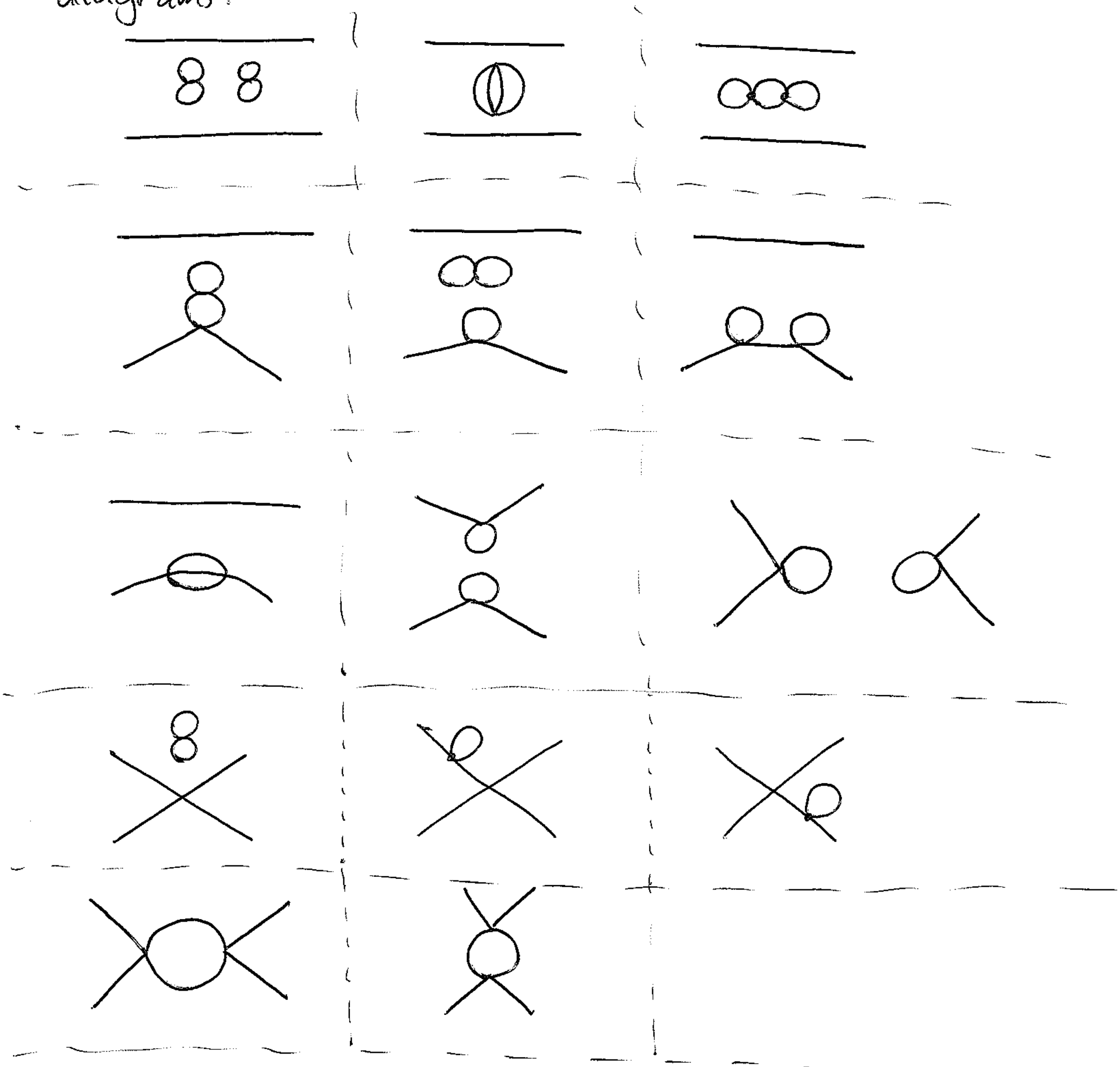
- 8: # ways line 1 can connect with a +
- 4: # ways line 2 can connect w. the other +
- 6: # ways line 3 can connect with one of the remaining lines in a +
- 3: # of ways line 4 can connect w. remaining lines on the other +
- 2: # of ways to join up the remaining lines on the two +'s.

Note that

this:  is the same as

this  since there is no distinction (in this case) between the two creation operators.

For the amplitude above, we have the following diagrams:



The result for the full expectation value is the sum

$$\sum_{\text{diagrams}} (\#\text{ ways to get each diagram}) \times (\text{product of 2pt functions corresponding to that diagram})$$

Fortunately, for most field theory calculations, we will be able to ignore all but the last two (generally we'll see that disconnected diagrams don't contribute to the things we want to calculate).