WORKSHEET

1) Suppose we want to find a real solution to

$$x^5 + \frac{x}{100} - 1 = 0$$

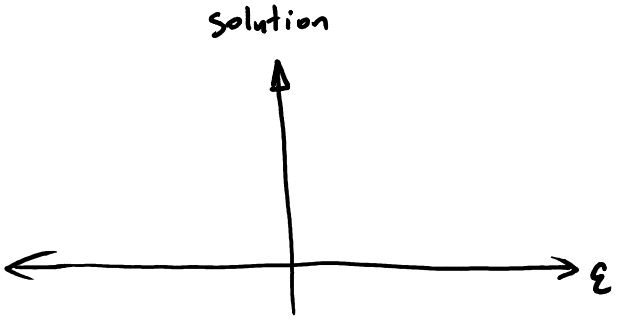
with no calculator or computer. Can you argue that there is a solution? Can you argue that there is only one solution? Can you guess what the solution is approximately equal to? Hint: you may want to sketch a graph of the function on the left side.

② Now consider the more general problem

$$x^5 + \varepsilon x - 1 = 0$$

a) Is there some ε for which you can find an exact solution easily?

b) If the solution for ε close to this value is $x(\varepsilon)$, draw a graph of what $x(\varepsilon)$ might look like near this value of ε.



c) If we are very close to the easy value for ε, what would likely be a useful mathematical approximation to the function $x(\varepsilon)$?

③ Hopefully, you ended up with something like:

$$x \approx 1 + c_1 \varepsilon + \ldots \qquad ①$$

Now, this is supposed to be a solution to

$$x^5 + \varepsilon x - 1 = 0 \qquad ②$$

If we substitute ① into ②, we can rearrange the terms to get an equation like: $(\quad) + \varepsilon(\quad) + \varepsilon^2(\quad) + \ldots = 0$

a) What are the terms in the 1st and 2nd set of brackets?

b) This is supposed to hold for each value of ε. Can you argue that each term in brackets must vanish?

c) Using b), what is c_1?

d) In the original problem, $\varepsilon = \frac{1}{100}$. If we just keep the terms up to order ε in the solution, what approximation do we get for the answer?

e) How can you estimate the accuracy of this?

f) Can you find the solution up to order ε^2?