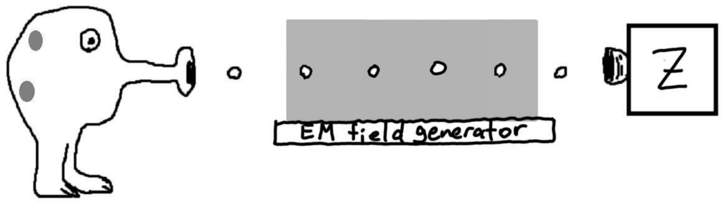
Physics 402 worksheet

Please work in groups of ~ 3 people, but write on your own worksheet since this will be part of your homework.



Particles in the state $|Z = 1\rangle$ are produced and travel through an electromagnetic field such that their state evolves via a Hamiltonian which is represented in the basis of Z eigenstates as

$$\hat{H} = E_0 \begin{pmatrix} 3 & 1 \\ 1 & 3 \end{pmatrix} . \tag{1}$$

After traveling through the electromagnetic field for a time T, the particle enters a Z detector. What is the probability that the particle will be measured in the state $Z = 1$?

Hint: you have seen this matrix before.

Step 1: find eigenvectors & eigenvalues for H

For matrix $\begin{pmatrix} 3 & 1 \\ 1 & 3 \end{pmatrix}$, characteristic polynomial is $\lambda^2 - 6\lambda + 8 = (\lambda-4)(\lambda-2)$

Corresponding normalized eigenvectors (using $\begin{pmatrix} b \\ \lambda - a \end{pmatrix}$ trick) are $\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ for $\lambda = 4$ and $\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$ for $\lambda = 2$. Eigenvectors for $\hat{H}$ are the same, with the eigenvalues multiplied by E_0

Step 2: Write $|\Psi(0)\rangle$ in terms of these eigenvectors:

In Z basis, $|Z=1\rangle$ represented as

$$\underbrace{\begin{pmatrix} 1 \\ 0 \end{pmatrix}}_{|Z=1\rangle} = \frac{1}{\sqrt{2}}\left[\underbrace{\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}}_{|E=4E_0\rangle} + \underbrace{\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix}}_{|E=2E_0\rangle}\right]$$

Step 3: Time evolve: $|E_n\rangle \rightarrow e^{-iE_nT/\hbar}|E_n\rangle$ so:

$$|\Psi(t)\rangle = \frac{1}{\sqrt{2}}\left[\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix} \cdot e^{-\frac{4iE_0T}{\hbar}} + \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{-\frac{2iE_0T}{\hbar}}\right]$$

$$P_{Z=1}(T) = |\langle Z=1|\Psi(t)\rangle|^2 = \left|\frac{1}{2}\left(e^{-4iE_0T/\hbar} + e^{-2iE_0T/\hbar}\right)\right|^2$$

$$= \cos^2\left(\frac{E_0T}{\hbar}\right)$$

Homework Problem

The internal states of a particle of spin 1 are described by a three-dimensional Hilbert space. We can choose a basis of eigenstates of and operator S_z with eigenvalues (in units of $\hbar$) -1, 0, and 1. Suppose the Hamiltonian for this system (perhaps coming from the interaction of this spin with a magnetic field) is represented in this basis by

$$H = E_0 \begin{pmatrix} 9 & 0 & 0 \\ 0 & 10 & -4i \\ 0 & 4i & 4 \end{pmatrix} . \tag{2}$$

where $E_0 = 10^7 s^{-1} \cdot \hbar$. So for example, $\hat{H}|-1\rangle = 9E_0|-1\rangle$. If the state at $t = 0$ is $\frac{1}{\sqrt{2}}(|-1\rangle + |1\rangle)$ what is the probability that the spin S_z will be measured to be 0 at some later time $T = 10^{-8}s$?