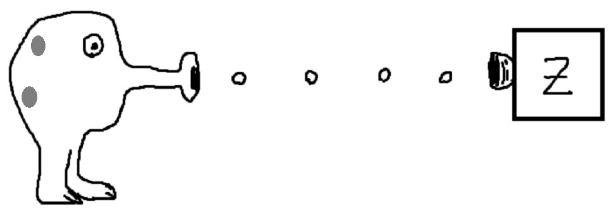
Physics 402 worksheet

Please work in groups of 3 people, but you will probably want to take your own notes since this will be part of your homework.

Question 1



You run the Q-zoo. It's not just any old zoo. At the Q-zoo, you have a collection of special animals (Qmu) that each expel particles with some internal states in a Hilbert space whose dimension is the same as the number of spots on the animal. All of the particles emitted by a particular Qmu are in precisely the same quantum state, which doesn't change unless the particle is measured. The animals all look pretty much identical, but it's very important to be able to tell them apart, because animals with the same number of spots that emit particles whose states are too close to being orthogonal will kill each other.

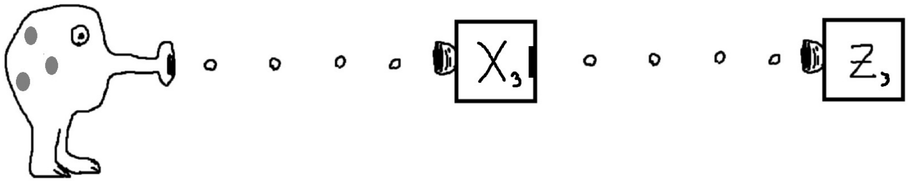
Your job this morning is to identify the quantum state associated with a particular two-spotted Qmu, which you can find using the link that I've placed at the bottom of our course webpage.

In order to identify the animals, you have detectors that each measure some particular observable associated with the particles. The Z detector measures an observable associated with a Hermitian operator whose matrix elements in a specific basis are $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$. The X detector measures an observable X associated with a Hermitian operator whose matrix elements in the same basis as before are $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$. Finally, the Y detector measures an observable associated with the operator with matrix elements $\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$.

Give an expression for the quantum state of these particles in terms of the eigenstates $|1\rangle$ and $|-1\rangle$ of Z.

Hint: can you figure out $\langle Z \rangle$? What does this tell you about the state? Ask for more hints if you are stuck.

Question 2



Next up, it's time for the monthly recalibration of the detectors for three-spotted Qmu. You bring out two detectors X_3 and Z_3, each of which measure observables whose possible values are -1, 0, and 1. In a basis $|-1\rangle$, $|0\rangle$, and $|1\rangle$ of eigenstates of Z_3, the operator associated with observable X_3 acts as

$$|-1\rangle \to \frac{\sqrt{2}}{2}|0\rangle \qquad |0\rangle \to \frac{\sqrt{2}}{2}|1\rangle + \frac{\sqrt{2}}{2}|-1\rangle \qquad |1\rangle \to \frac{\sqrt{2}}{2}|0\rangle$$

To check the detectors, you bring out a particular three-spotted Qmu named JoJo whose particles are in an equal magnitude superposition of the three possible eigenstates for X_3. You measure Jojo's particles with the X_3 detector but have them pass through to be measured by the Z_3 detector. If the detectors are working correctly, what results should you find for the following table?

X measurement	% with Z=-1	% with Z=0	% with Z=1
-1			
0			
1			