LAST TIME: $H = H_0 + H'(t)$ $\quad |\psi(t=t_0)\rangle = |\psi_a\rangle$

turns on at $t = t_0$

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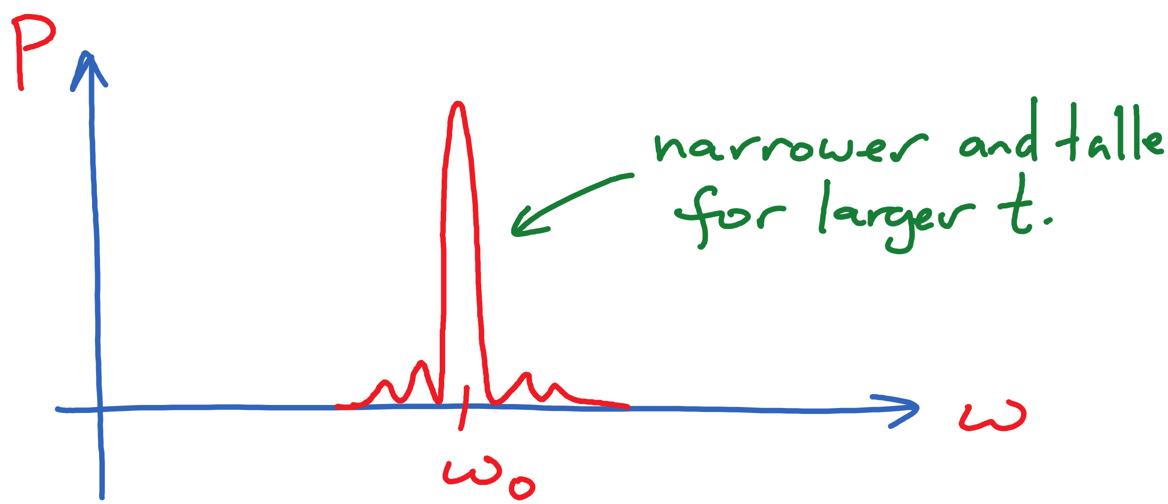
Videos: ① Sinusoidal time dependence $H'_{ba}(t) = V\cos(\omega t)$

Gives $P_{a\to b}(t) \approx \dfrac{|V_{ba}|^2}{\hbar^2}\,\dfrac{\sin^2((\omega-\omega_0)t/2)}{(\omega-\omega_0)^2}$

fixed time:



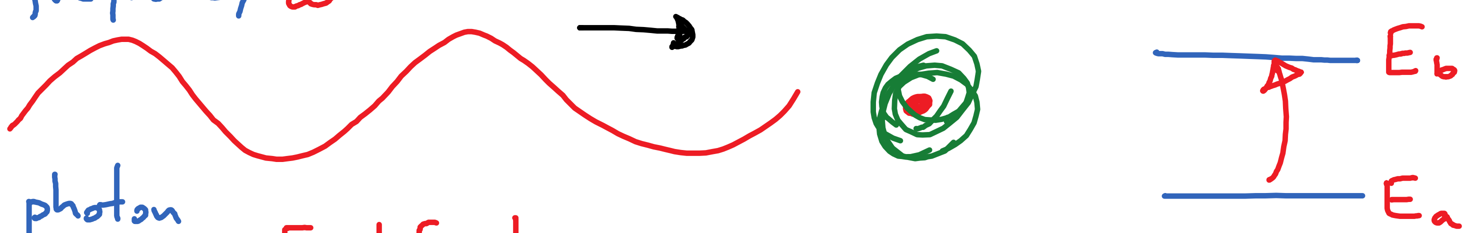
Q: A harmonic oscillator with frequency ω_0 is in the state $|3\rangle$. If we add a perturbation $A\hat{x}^2\cos(\omega t)$ the transition probability to the state $|1\rangle$ after some time will be largest for ω equal to

$$\omega = \frac{|E_1 - E_3|}{\hbar} = \frac{2\hbar\omega_0}{\hbar} = 2\omega_0$$

1. ω_0
2. $2\omega_0$
3. $3\omega_0$
4. $4\omega_0$
5. None of these: the system won't make a transition to a lower energy state

Next step: Apply to atomic transitions:

frequency ω



photon energy $E = hf = \hbar\omega$

energy difference $\Delta E = \hbar\omega_0$

$\omega \approx \omega_0$: photon energy matches energy difference

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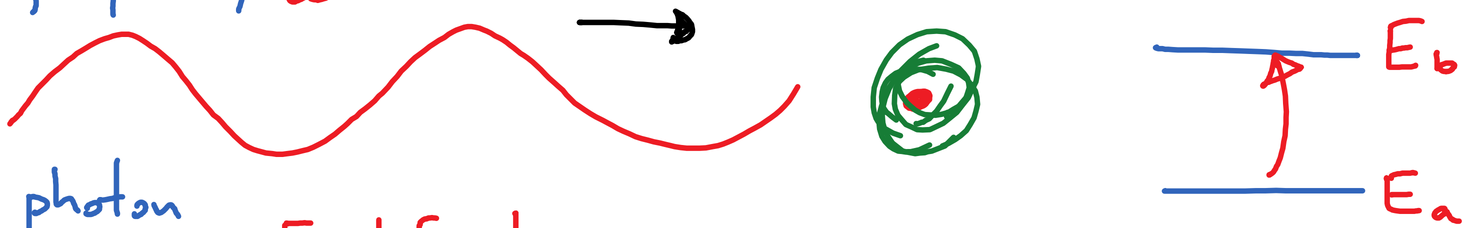

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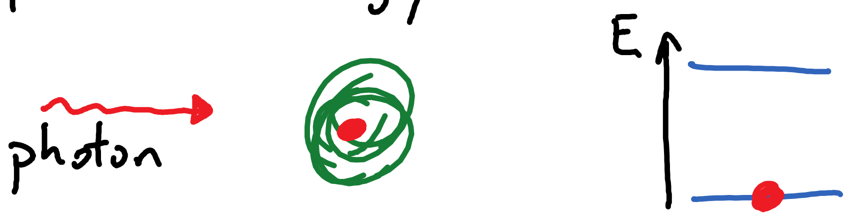
BEFORE:

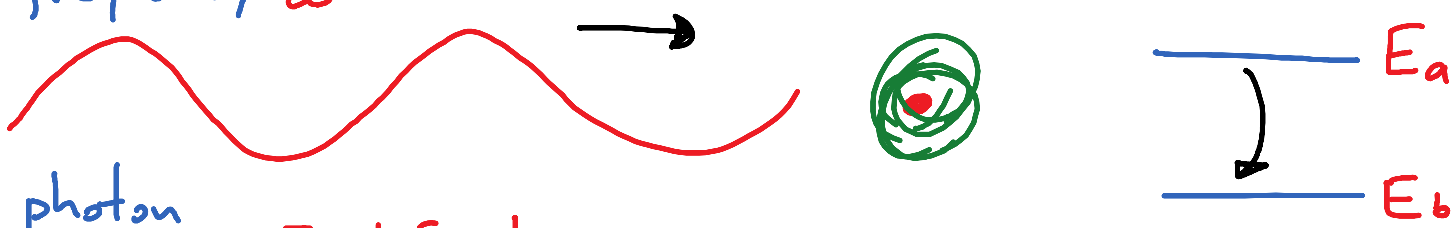


ABSORPTION

AFTER:

Next step: Apply to atomic transitions:

frequency ω



photon energy $E = hf = \hbar\omega$

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BEFORE:

AFTER:

STIMULATED EMISSION

Hamiltonian for charged particle in electromagnetic field:

$$\vec{E}(x,t),\ \vec{B}(x,t)$$

$$\Downarrow$$

$$\vec{A}(x,t),\ \phi(x,t) \quad \text{s.t.} \quad \begin{aligned} \vec{\nabla}\times\vec{A} &= \vec{B} \\ -\vec{\nabla}\phi - \partial_t\vec{A} &= \vec{E} \end{aligned}$$

$$\Downarrow$$

$$\Delta H_{EM} = q\,\phi(\vec{x},t) - \frac{q}{m}\vec{p}\cdot\vec{A}(x,t) + \frac{q^2}{2m}\vec{A}^2(x,t)$$

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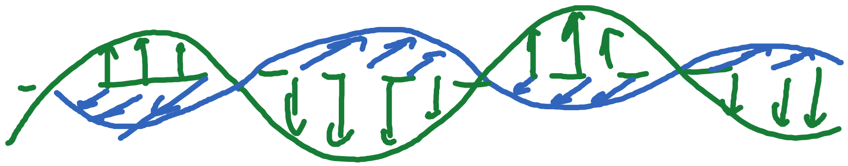
e.g. $\vec{B} = B\hat{z}$ $\quad \vec{E} = 0$

see HW11 solutions



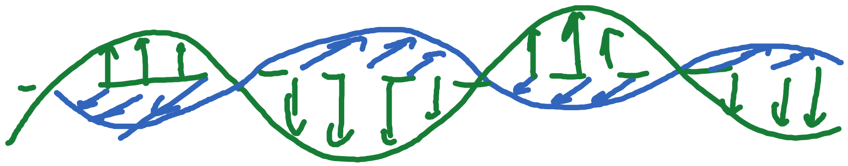
For $\vec{E}(x,t) = \vec{E}_0 \cos(\vec{k}\cdot\vec{x} - \omega t)$, $\vec{B}(x,t) = \vec{B}_0 \cos(\vec{k}\cdot\vec{x} - \omega t)$

$|\vec{k}| = \frac{2\pi}{\lambda}$

Q: What is $\dfrac{\lambda_{visible}}{\text{size of atom}}$ (order of magnitude)

1. 0.5
2. 50
3. 5000
4. 500,000

$$\frac{500\,nm}{1\,\mathring{A}} = \frac{5000\,\mathring{A}}{1\,\mathring{A}} = 5000$$



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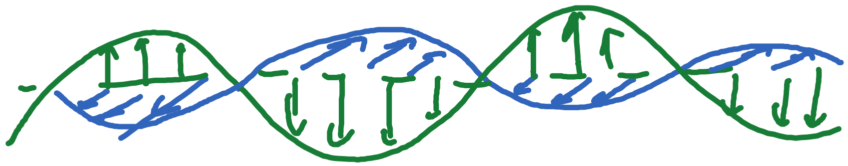
$|\vec{k}| = \frac{2\pi}{\lambda}$

Have $|\vec{k}\cdot\vec{x}| \ll 1$. To a good approx. can ignore and model EM wave as $\vec{E}(t) = \vec{E}_0 \cos(\omega t)$. Effects of $\vec{B}$ also suppressed (see video for much more careful treatment).



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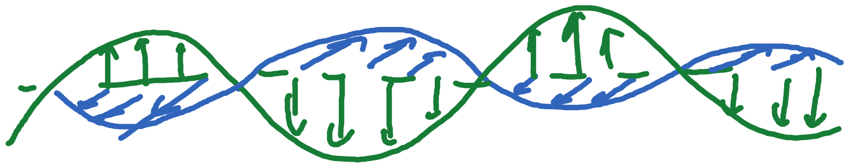
Result: can take $\phi \approx -\vec{E}_0 \cdot \vec{x} \cos(\omega t)$ $\vec{A} \approx 0$



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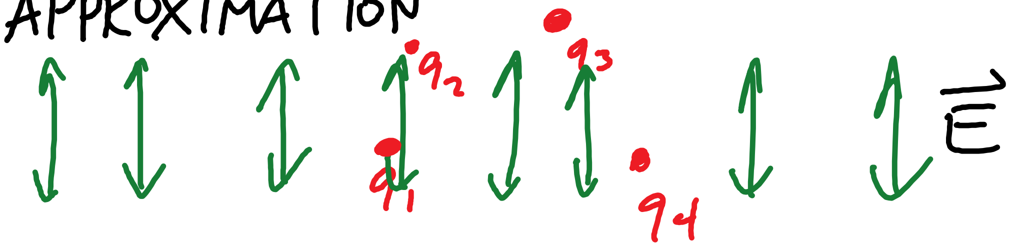
Result: can take $\phi \approx -\vec{E}_0 \cdot \vec{x} \cos(\omega t)$ $\quad \vec{A} \approx 0$

$$\Delta H_{EM} \approx q\,\phi = -q\vec{E}_0 \cdot \vec{x} \cos(\omega t)$$

The DIPOLE APPROXIMATION



$$\Delta H_{EM} = -q\vec{E}_0 \cdot \vec{x} \cos(\omega t)$$ for each charge

Total: $$\Delta H_{EM} = -\vec{E}_0 \cdot \vec{P} \cos(\omega t)$$

$$\vec{P} = \sum_i q_i \vec{x}_i$$ Dipole moment operator

Transition Rates:

Start with $\Delta H_{EM} = \underbrace{-\vec{E}_0 \cdot \vec{P}}_{V} \cos(\omega t)$

Plug in to $P_{a\to b} = \dfrac{|V_{ab}|^2}{\hbar^2} \dfrac{\sin^2((\omega-\omega_0)t/2)}{(\omega-\omega_0)^2}$ $(*)$

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Real radiation has range of frequencies. Define:

$\rho(\omega)d\omega$ energy density in range $[\omega, \omega+d\omega]$

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Add transition probability from all frequencies, average over directions + polarizations of radiation.

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Plug into $P_{a\to b} = \dfrac{|V_{ab}|^2}{\hbar^2} \dfrac{\sin^2((\omega-\omega_0)t/2)}{(\omega-\omega_0)^2}$ (*)

Real radiation has range of frequencies. Define:

$\rho(\omega)d\omega$ energy density in range $[\omega, \omega+d\omega]$

Add transition probability from all frequencies, average over directions + polarizations of radiation.

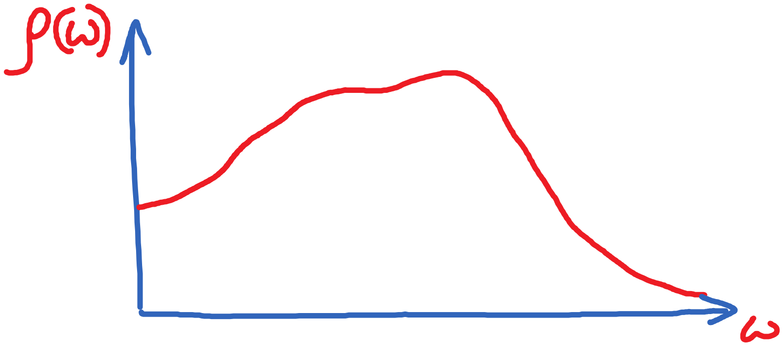
Final result: $P_{a\to b} = R_{a\to b} \cdot t$

TRANSITION RATE:

$$R_{a\to b} = \frac{\pi}{3\varepsilon_0 \hbar^2} \left|\vec{\mathcal{P}}_{ba}\right|^2 \rho(\omega_{ba})$$

$$R_{a\to b} = \frac{\pi}{3\varepsilon_0 \hbar^2} \left| \vec{\wp}_{ba} \right|^2 \rho(\omega_{ba})$$

Q : A hydrogen atom in the $|100\rangle$ state is in a bath of radiation with energy density shown. What is the transition rate to the $|211\rangle$ state?



$$R_{a\to b} = \frac{\pi}{3\varepsilon_0 \hbar^2} \left| \vec{\mathcal{P}}_{ba} \right|^2 \rho(\omega_{ba})$$

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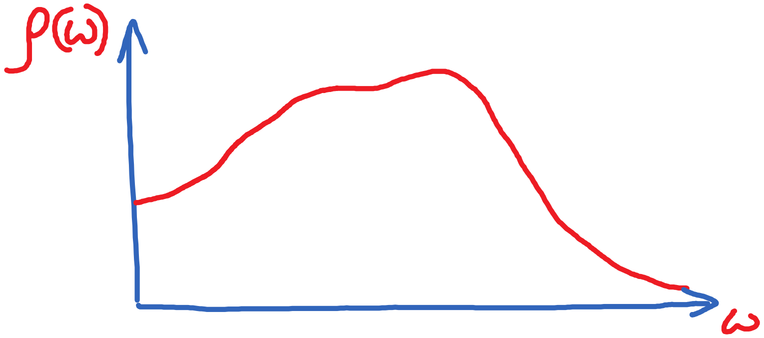

DISCUSSION QUESTIONS: What is $\left|\vec{\mathcal{P}}_{ba}\right|^2$ here? Write an expression in terms of the wavefunctions $\psi_{100}(\vec{x})$ and $\psi_{211}(\vec{x})$

$$\vec{\mathcal{P}}_{ba} = \int d^3x \, \psi^*_{211}(\vec{x}) \, (-e\vec{x}) \, \psi_{100}(\vec{x})$$

How do we use the information in the graph? evaluate ρ at $\frac{|E_2 - E_1|}{\hbar}$