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$H'(t)$ turns on at time t_0

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Probability of measuring particle in state $|\psi_b\rangle$ at time t

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$$\omega_{ba} = \frac{E_b - E_a}{\hbar} \qquad\qquad H'_{ba}(t) = \langle\psi_b|H'(t)|\psi_a\rangle$$

From 2019 exam:

A system of two spins has a Hamiltonian $H = \frac{\Omega}{\hbar} J^2$ where $\vec{J} = \vec{S}_1 + \vec{S}_2$. At $t = 0$, the system is in the state

$$|J = 1, M = 0\rangle = \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle \right) . \tag{4}$$

Starting at $t = 0$, a time-dependent perturbation

$$H_1 = A e^{-bt} S_2^z \tag{5}$$

is added to the Hamiltonian, where $S_2^z = \mathbb{1} \otimes S^z$ is the z-component of the spin for the second particle.

a) In the approximation of first order time-dependent perturbation theory, what is the probability that the system will have made a transition to the $J = 0$ state

$$|J = 0, M = 0\rangle = \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) . \tag{6}$$

at time $t = \infty$? **(4 points)**

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Q1: what are t_0 and t?

1. $-\infty$ and ∞
2. $-\infty$ and 0
3. 0 and ∞
4. 0 and t
5. $-\infty$ and b

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Q2: what is ω_{ba}?

Q3: what is $H'_{ba}(t)$?

(Discussion)

$\omega_{ba} = (E_b - E_a)/\hbar$

$$H_0 |\psi_a\rangle = \frac{\Omega}{\hbar} J^2 |J=1, M=0\rangle$$
$$= \frac{\Omega}{\hbar} \cdot \hbar^2 \cdot 1 \cdot 2 \cdot |J=1, M=0\rangle$$
so $E_a = 2\Omega\hbar$

$$H_0 |\psi_b\rangle = \frac{\Omega}{\hbar} J^2 |J=0, M=0\rangle$$
$= 0$ so $E_b = 0$

$$H'_{ba}(t) = \frac{1}{\sqrt{2}} (\langle\uparrow\downarrow| - \langle\downarrow\uparrow|) \cdot A e^{-bt} S_2^z \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)$$
$$= A \hbar_{/2} e^{-bt}$$

$= 0 \quad \text{so } E_6 = 0$

Consider a quantum system described by a Hamiltonian

$$H = \frac{p^2}{2m} + \frac{1}{2}k(t)x^2 \ ,$$

where

$$k(t) = m\omega_0^2 + \frac{K}{1 + \left(\frac{t}{\tau_0}\right)^2} \ .$$

a) At $t = -\infty$, the system starts in the ground state of the Hamiltonian at that time. For small values of the parameter K, estimate the probability that the system will have made a transition to some other state, by time $t = \infty$.

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Q_1: What is H_0?

$H_0 = H$ at $t=-\infty$

① $\frac{p^2}{2m}$

② $\boxed{\frac{p^2}{2m} + \frac{1}{2}m\omega_0^2 x^2}$

③ $\frac{p^2}{2m} + \frac{1}{2}(m\omega_0^2 + K)x^2$

④ $\frac{p^2}{2m} + \frac{1}{2}\left(m\omega_0^2 + \frac{K}{1+(t/\tau_0)^2}\right)x^2$

⑤ $\varnothing$

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Q2: What is $H'(t)$?

① $\frac{1}{2}m\omega_0^2x^2 + \frac{1}{2}\frac{K}{1+\left(\frac{t}{\tau_0}\right)^2}x^2$

② $\frac{1}{2}\frac{K}{1+\left(\frac{t}{\tau_0}\right)^2}x^2$

③ $\frac{p^2}{2m} + \frac{1}{2}m\omega_0^2x^2$

④ $-\frac{t x^2 K}{\left(1+\left(\frac{t}{\tau_0}\right)^2\right)^2}$

⑤ $\varnothing$

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Q3: The system starts in the state $|0\rangle$. To which other states can we make a transition (in the approximation of 1st order perturbation theory?)

Hint: what is H'_{ba}?

(DISCUSSION)

$$H'_{ba} = \langle n | \frac{1}{2} \frac{K}{1+\left(\frac{t}{\tau_0}\right)^2} x^2 | 0 \rangle$$

→ write as $(a+a^\dagger)^2 \frac{\hbar}{2m\omega}$

only $n=2$ possible

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a) At $t = -\infty$, the system starts in the ground state of the Hamiltonian at that time. For small values of the parameter K, estimate the probability that the system will have made a transition to some other state.

Q4: What is ω_{ba}?

$$\frac{E_2 - E_0}{\hbar} = \frac{2\hbar\omega_0}{\hbar}$$

$$= 2\omega_0$$

① ω_0

② $\frac{1}{2}\omega_0$

③ $2\omega_0$

④ $\varnothing$

Problem 3

A particle of mass m is in the ground state of an infinite square well potential of width a. Starting at $t = 0$, the potential in the left half of the well increases at a constant rate from 0 to V in time T and then decreases back to zero at a constant rate in time T. If V is small, what is the probability that the particle will be found in the first excited state of the well at time $2T$? *Hint: the formula sheet should help.*

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Q: What is $H'_{ba}(t)$?

* Useful formulae: $\psi_n(x) = \sqrt{\frac{2}{a}} \cdot \sin\left(\frac{n\pi x}{a}\right)$

(DISCUSSION)

$$\langle \psi_2 | H' | \psi_1 \rangle = \int dx\, \psi_2^*(x)\, V(x,t)\, \psi_1(x)$$

$$= \int_0^{a/2} dx\, \psi_2^*(x)\, \psi_1(x)\, V(t)$$

Videos ("reading" for Tuesday)

1. $P_{a\to b}$ for a sinusoidal time dependence
2. Types of atomic transitions
3. H for a charged particle in an EM field
4. $H'(t)$ for an atom/molecule in an EM wave