PHYSICS 402 WORKSHEET

(1) You are stranded on a desert island with nothing but your tattered clothing and a Sharp EL-510 calculator. Trying to fend off feelings of despair and hopelessness, you consider trying to estimate the number of sand grains on the island to within an order of magnitude, but your heart is not in it, and you soon find yourself staring blankly out at the apparently boundless ocean. Unconsciously, you reach down to scratch one of the festering sores on your ankle and by sheer coincidence, your hand brushes against a larger-than-average shell that happens to be the summer home of a less-generous-than-average Genie. Bound by the standard contractual obligations, the Genie materializes from the shell, looks at you with a mix of irritation and disgust, and reluctantly announces "Okay, let's get this over with. I will grant you one wish. You can't wish for more wishes. And first, you'll have to answer a skill-testing question..."

"What is the minimum value of the function

$$f(x,y,z) = \sqrt{\log(x^4+2y^2+9) + (y-7x^3)^4 + 12(xy-3)^2}$$

for real x and y, to 5% accuracy?"

What would you do to answer the question?

Could just try lots of values.

For any x, y, z,

$$f_{min} \leq f(x,y,z)$$

e.g. $x=y=0$: $f_{min} \leq \sqrt{\log(9)+108}$

If we try enough values, we'll eventually be confident that our lowest one is close to the actual min.

② Suppose H is a Hamiltonian for a quantum system with a 5 dimensional Hilbert space. The matrix representation for H is:

$$\begin{pmatrix} 3 & 1 & 6 & 5 & 2 \\ 1 & 9 & 2 & 4 & 8 \\ 6 & 2 & 1 & 6 & 7 \\ 5 & 4 & 6 & 4 & 0 \\ 2 & 8 & 7 & 0 & 5 \end{pmatrix}$$

a) Can you figure out what the ground state energy is? Yes but need to solve quintic char. poly.. Can't do it by hand.

b) For the state represented by $\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$, what is $\langle \Psi | H | \Psi \rangle$?

$= (1\,0\,0\,0\,0) \cdot \left(H_{5\times 5} \right) \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} = 3$

c) Can you say anything about how your answer to b) compares to the ground state energy? $E_{g.s.} \leq 3$ since $\langle \Psi | H | \Psi \rangle$ is an avg. of measurements of energy

d) Can you come up with a better upper bound E_0? Use $\begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} \rightarrow E_{g.s} \leq 1$

③ Suppose we are given a Hamiltonian H for some quantum system. Based on your answer to ②, can you think of a general method that could be used to place an upper bound on the ground state energy?

See first variational method video.