WORKSHEET: Hyperfine splitting

For the $n=1$ level of hydrogen, taking into account the electron spin and the proton spin, we have four basis states in the S_z^e, S_z^p basis. We can write these as:

$$|s_z^e\rangle \otimes |s_z^p\rangle$$

where $s_z^e = \pm\frac{1}{2}$ and $s_z^p = \pm\frac{1}{2}$. The interaction between the two magnetic moments gives a perturbation to the Hamiltonian that for the $n=0$ level is equivalent to:

$$H_1 = C\,\vec{S}^e \cdot \vec{S}^p \qquad (C \text{ is a positive constant}).$$

Taking into account H_1 as a perturbation, determine the energy shifts at first order in perturbation theory and write the energy eigenstates. (Hint: you can use the same trick we used for the spin-orbit coupling)