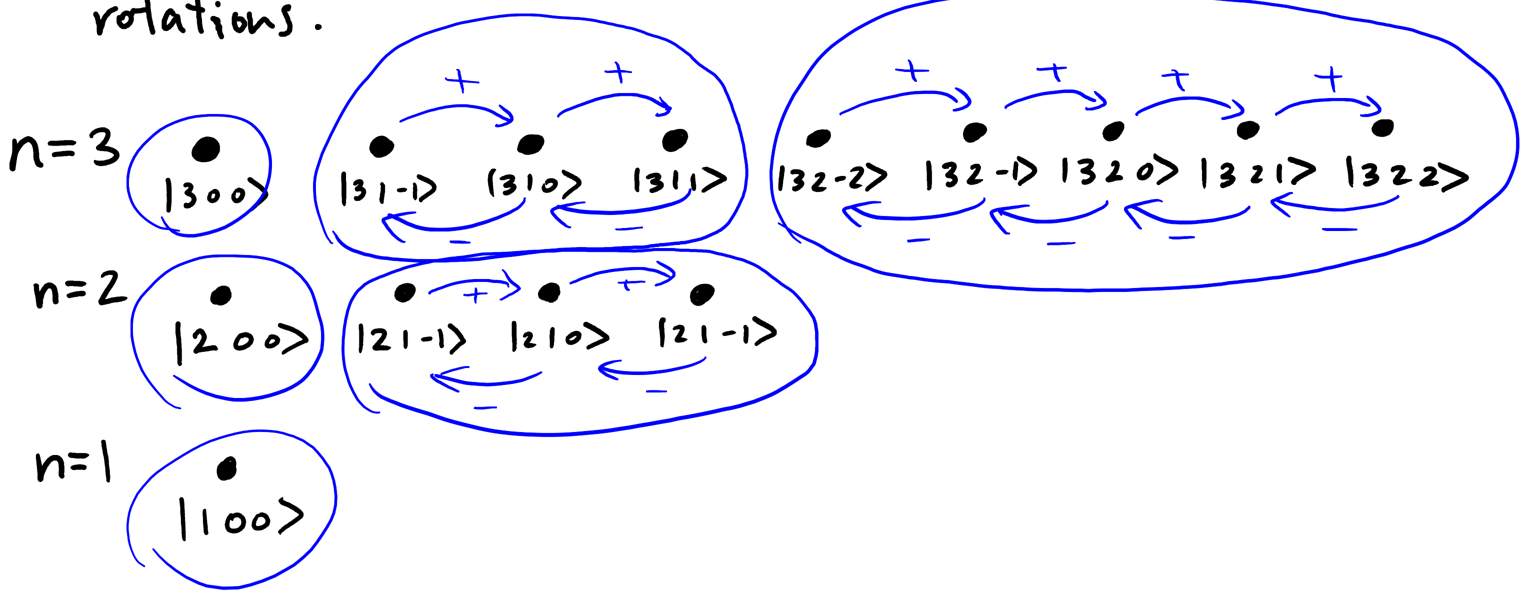
PHYSICS 402 WORKSHEET

① The following chart shows some low-energy states of the hydrogen atom (before considering the electron spin) in the $|n\,\ell\,m\rangle$ notation. Show how L_+ and L_- act on each of the states by drawing $\xrightarrow{+}$ and $\xrightarrow{-}$ arrows. Circle the groups that can be transformed into one another by rotations.



② Write the results of the following:

a) $L_z|2\,1\,1\rangle = \hbar|2\,1\,1\rangle$

b) $L^2|2\,1\,1\rangle = \hbar^2\cdot 2\cdot|2\,1\,1\rangle$

c) $L_+|2\,1\,1\rangle = 0$

d) $L_-|2\,1\,1\rangle = \sqrt{2}\hbar|2\,1\,0\rangle$

e) $L_x|2\,1\,1\rangle = \frac{1}{2}(L_+ + L_-)|2\,1\,1\rangle = \frac{\sqrt{2}}{2}\hbar|2\,1\,0\rangle$

extra: $L_xL_y|2\,1\,0\rangle$

$= \frac{1}{2}(L_+ + L_-)\cdot\frac{-i}{2}(L_+ - L_-)|2\,1\,0\rangle = -\frac{i}{4}(L_-L_+ + L_+L_-)|2\,1\,0\rangle = -i|2\,1\,0\rangle$

③ For states of a spin 1 particle of the form

$$\sum_M c_M |M\rangle ,$$

which are physically unchanged after a rotation about the z axis?

What about the x-axis?

Want $|\Psi\rangle \to e^{i\alpha} |\Psi\rangle$

Generally: $|\Psi\rangle \to e^{-\frac{i}{\hbar}\theta J_z} |\Psi\rangle$

Will be a simple phase times $|\Psi\rangle$ if and only if $|\Psi\rangle$ is an eigenstate of J_z:

$\therefore$ $|1\rangle$, $|0\rangle$, $|-1\rangle$ physically unchaged after rotation about z.

Rotation about x leaves state physically unchanged if eigen state of J_x

$\therefore$ solve: $\frac{1}{2}(J_+ + J_-)|\Psi\rangle = \hbar M |\Psi\rangle$