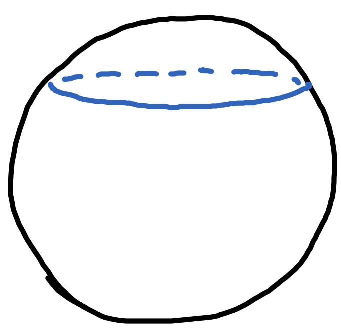
Worksheet solutions (partial)

① Let's call the eigenstates in the Z basis $|-1\rangle$ and $|1\rangle$. Then we can write the most general state as

$$|\Psi\rangle = z_1 |1\rangle + z_{-1} |-1\rangle$$

We can assume $|z_1|^2 + |z_{-1}|^2 = 1$ and that z_1 is real and positive. From the experiment, about 3/4 of the Z measurements are +1, so $P_1 \approx \frac{3}{4}$ and $P_{-1} = \frac{1}{4}$. This tells us $|z_1|^2 = 3/4$, $|z_2|^2 = \frac{1}{4}$, so $z_1 = \frac{\sqrt{3}}{2}$ (we assumed it's real & positive) and $z_2 = \frac{1}{2} e^{i\varphi}$, but we don't know what φ is yet.

Graphically, we have narrowed down the possible quantum state to a particular circle on the sphere that describes our most general state.



Next, we want to use the X and Y measurements to constrain the state further. From our state $|\psi\rangle = \frac{\sqrt{3}}{2}|1\rangle + \frac{1}{2}e^{i\varphi}|-1\rangle$, we can now compute either $\langle X \rangle$ or $P_{X=1}$ and $P_{X=-1}$ and compare with our experimental results.