MIDTERM PRACTICE ANSWERS

① z angular momentum L_z is conserved (also energy if H is time independent.

② 10^{-10} m

③ $E_{photon} = E_{n=2} - E_{n=1} = -\frac{13.6\,eV}{4} - (-13.6\,eV) \approx 10.9\,eV$

④ $[\hat{x}, \hat{p}] = i\hbar$

⑤ Can check if $\left|\frac{\delta E_n^1}{E_n^0}\right| \ll 1$ (and/or $\left|\frac{\delta E_n^2}{\delta E_n^1}\right| \ll 1$).

⑥ $|\Psi_2\rangle - |\Psi_1\rangle = -i\frac{\hat{P}}{\hbar}\epsilon\,|\Psi_1\rangle$

⑦ $\langle x|\hat{x}|y\rangle = y\,\langle x|y\rangle = y\,\delta(x-y)$ or $x\,\delta(x-y)$

⑧-⑬ see quiz 1 solutions

① If $\vec{\mu} = \alpha\vec{S}$ then $H = -\alpha B S_z$ and $P = \cos^2\left(\alpha T B \frac{\hbar}{2}\right)$

② a) $|2\,0\rangle, |1\,1\rangle, |0\,2\rangle$ in $|n_x\,n_y\rangle$ basis

b) $L_z = x p_y - y p_x$

c) write L_z in terms of $a_x, a_y, a_x^\dagger, a_y^\dagger$. Demand $L_z|\psi\rangle = 0$.

result: $\frac{1}{\sqrt{2}}(|2\,0\rangle + |0\,2\rangle) = \frac{1}{\sqrt{2}}(a_x^\dagger + a_y^\dagger)|0\,0\rangle$

③ a) Write $Q = \frac{1}{2}m\omega^2$. So $\omega = \sqrt{\frac{2Q}{m}}$. $E_0 = \frac{1}{2}\hbar\sqrt{\frac{2Q}{m}}$

b) $V(x) = Qx^2 + \lambda Q x^3 + \lambda^2 Q x^4 + \ldots$

c) Have $\Delta E = \lambda^2 Q\langle 0|x^4|0\rangle$ at 1st order

$$\Delta E = \lambda^2 \sum_{n\neq 0} \frac{|\langle n|x^3 Q|0\rangle|}{E_0 - E_n} \qquad E_n = \hbar\sqrt{\frac{2Q}{m}}\left(n+\frac{1}{2}\right)$$

4) a) Have basis $|\uparrow\uparrow\rangle$ $E = A\hbar$

$|\uparrow\downarrow\rangle$, $|\downarrow\uparrow\rangle$ $\Big\}$ $E = 0$ degeneracy 2

$|\downarrow\downarrow\rangle$ $E = -A\hbar$

use $S_x = \frac{\hbar}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

b) Add $cS_x^1 S_x^2$

$$|\uparrow\uparrow\rangle: \quad \delta E = \langle\uparrow\uparrow| c S_x^1 S_x^2 |\uparrow\uparrow\rangle$$
$$= c\langle\uparrow|S_x^1|\uparrow\rangle\langle\uparrow|S_x^1|\uparrow\rangle$$
$$= 0$$

$$|\downarrow\downarrow\rangle \quad \delta E = \langle\downarrow\downarrow| c S_x^1 S_x^2 |\downarrow\downarrow\rangle$$
$$= c\langle\downarrow|S_x^1|\downarrow\rangle\langle\downarrow|S_x^2|\downarrow\rangle$$
$$= 0$$

For $|\uparrow\downarrow\rangle$, $|\downarrow\uparrow\rangle$ use deg. p.T.. Matrix is:

$$\begin{pmatrix} \langle\uparrow\downarrow|cS_x^1 S_x^2|\uparrow\downarrow\rangle & \langle\uparrow\downarrow|cS_x^1 S_x^2|\downarrow\uparrow\rangle \\ \langle\downarrow\uparrow|cS_x^1 S_x^2|\uparrow\downarrow\rangle & \langle\downarrow\uparrow|cS_x^1 S_x^2|\downarrow\uparrow\rangle \end{pmatrix}$$

$$= \begin{pmatrix} 0 & c\left(\frac{\hbar}{2}\right)^2 \\ c\left(\frac{\hbar}{2}\right)^2 & 0 \end{pmatrix}$$

Eigenvalues: $c\left(\frac{\hbar}{2}\right)^2, -c\left(\frac{\hbar}{2}\right)^2$ = energy shifts.

5 a) $H = \frac{p_1^2}{2m} + \frac{p_2^2}{2m} + \frac{1}{2}m\omega^2 x_1^2 + 2m\omega^2 x_2^2$

b) $E = \hbar\omega(n_1+\frac{1}{2}) + 2\hbar\omega(n_2+\frac{1}{2})$. Energy $\frac{3}{2}\hbar\omega : |0\,0\rangle$

Energy $\frac{5}{2}\hbar\omega : |1\,0\rangle$ Energy $\frac{7}{2}\hbar\omega : |2\,0\rangle, |0,1\rangle$

Energy $\frac{9}{2}\hbar\omega : |3\,0\rangle, |1\,1\rangle$ Energy $\frac{11}{2}\hbar\omega : |4\,0\rangle, |2\,1\rangle, |0\,2\rangle$

b) $\delta E_0 = \langle 0\,0 | \lambda x_1^2 x_2 | 0\,0 \rangle = \lambda \langle 0 | x_1^2 | 0 \rangle \langle 0 | x_2 | 0 \rangle = 0$

c) Have matrix:

$$\begin{pmatrix} \langle 2\,0 | \lambda x_1^2 x_2 | 2\,0 \rangle & \langle 2\,0 | \lambda x_1^2 x_2 | 0\,1 \rangle \\ \langle 0\,1 | \lambda x_1^2 x_2 | 2\,0 \rangle & \langle 0\,1 | \lambda x_1^2 x_2 | 0\,1 \rangle \end{pmatrix}$$

$$= \begin{pmatrix} 0 & \lambda \langle 2 | x_1^2 | 0 \rangle \langle 0 | x_2 | 1 \rangle \\ \lambda \langle 0 | x_1^2 | 2 \rangle \langle 1 | x_2 | 0 \rangle & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \cdot \frac{\sqrt{2}}{4} \left(\frac{\hbar}{m\omega} \right)^{\frac{3}{2}}$$

$\frac{1}{\sqrt{2}}(|0\,1\rangle + |2\,0\rangle)$ shift $\frac{\sqrt{2}}{4} \left(\frac{\hbar}{\omega m} \right)^{\frac{3}{2}}$

$\frac{1}{\sqrt{2}}(|0\,1\rangle - |2\,0\rangle)$ shift $-\frac{\sqrt{2}}{4} \left(\frac{\hbar}{\omega m} \right)^{\frac{3}{2}}$

7. Have $\frac{d}{dt} \langle \psi | \hat{O} | \psi \rangle$

$$= \frac{i}{\hbar} \langle \psi | \hat{H} \hat{O} | \psi \rangle - \frac{i}{\hbar} \langle \psi | \hat{O} \hat{H} | \psi \rangle$$

$$= \frac{i}{\hbar} \langle \psi | [\hat{H}, \hat{O}] | \psi \rangle = 0$$