LAST TIME: Operators: linear map states $\rightarrow$ states

Given a basis $|\lambda_n\rangle$ can represent $\hat{a}$ by matrix $a_{mn} = \langle \lambda_m | \hat{a} | \lambda_n \rangle$

An eigenvector $|v\rangle$ for $\hat{a}$ with eigenvalue λ satisfies

$$\hat{a}|v\rangle = \lambda |v\rangle$$

Given observable a, have orthonormal basis of states $|\lambda_n\rangle$ with definite value λ_n for a. Define operator $\hat{a}$ by:

$$\hat{a}\,|\lambda_n\rangle = \lambda_n\,|\lambda_n\rangle.$$

In λ_n basis a_{mn} is diagonal: $\begin{pmatrix} \lambda_1 & & & \\ & \lambda_2 & & \\ & & \ddots & \\ & & & \lambda_n \end{pmatrix}$

Special, since $|\lambda_n\rangle$ orthogonal, λ_n real

$\Updownarrow$ MATH THEOREM

true if and only if $\hat{a}$ is HERMITIAN

$$a^*_{nm} = a_{mn}$$

matrix elements are a Hermitian matrix

$$\hat{a}^\dagger = \hat{a}$$

where ADJOINT $\hat{a}^\dagger$ defined by

$$\langle \psi_1 | \hat{a}^\dagger | \psi_2 \rangle = \langle \psi_2 | \hat{a} | \psi_1 \rangle^*$$

$\boxed{\text{OBSERVABLES} \longleftrightarrow \text{HERMITIAN OPERATORS}}$

How do we interpret $\hat{a}|\Psi\rangle$? <u>NEXT WEEK</u>

Useful fact: $\langle\Psi|\hat{a}|\Psi\rangle = \langle a\rangle$

PROOF: in a basis:

$$(c_1^* \cdots c_N^*)\begin{pmatrix}\lambda_1 & & \\ & \ddots & \\ & & \lambda_N\end{pmatrix}\begin{pmatrix}c_1\\ \vdots \\ c_N\end{pmatrix} = \sum_n |c_n|^2\lambda_n$$

$$= \sum_n P_n \lambda_n$$

$$= \langle a\rangle$$

Recall: the EXPECTATION VALUE $\langle a\rangle$ of an observable a is the avg. value over a large # of measurements: $\langle a\rangle = \sum_n \lambda_n \cdot P(\lambda_n)$

~ average

The UNCERTAINTY ΔO of an observable O in a state $|\Psi\rangle$ is defined by saying that $(\Delta O)^2$ is the expected value of $(O - \langle O\rangle)^2$.

~ standard deviation.