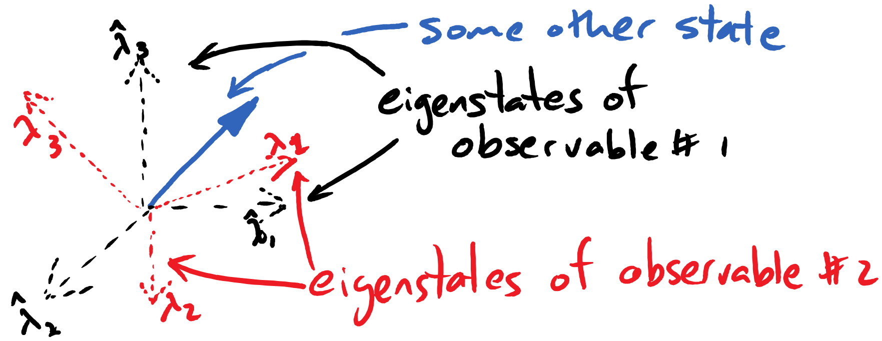
So far: Hilbert space



Characters so far: states $|\psi\rangle \longleftrightarrow$ vectors

observables $\longleftrightarrow$ orthonormal basis of states with definite value

"dual vectors": given $|\lambda\rangle$, define $\langle\lambda|$ as a linear map from states $\to$ complex numbers

$|\psi\rangle \to \langle\lambda|\psi\rangle \longleftarrow$ |square| gives probability of measuring λ

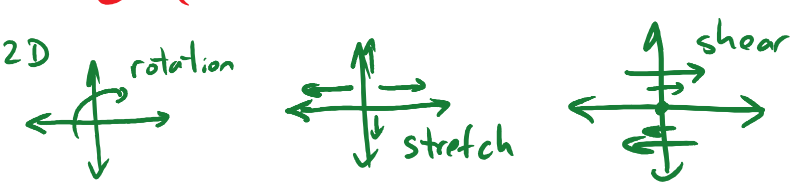
TODAY: OPERATORS = maps from states to states

essential to describe time evolution, symmetries, understand connection between symmetries + conserved quantities ...

Define (LINEAR) OPERATOR $\hat{a}$ = map from states $\longrightarrow$ states

linear: $\hat{a}(z_1|\psi_1\rangle + z_2|\psi_2\rangle) = z_1(\hat{a}|\psi_1\rangle) + z_2(\hat{a}|\psi_2\rangle)$

e.g. 2D



$\Rightarrow \hat{a}$ completely determined by action on basis

$$\hat{a}|\lambda_n\rangle = \sum_m a_{mn}|\lambda_m\rangle$$

represented by a MATRIX

For $|\psi\rangle = \sum c_n|\lambda_n\rangle$, $\hat{a}|\psi\rangle = \sum c_n \hat{a}|\lambda_n\rangle$

$a_{mn} = \langle\lambda_m|\hat{a}|\lambda_n\rangle$ are the matrix elements of a in basis $|\lambda_n\rangle$

To understand behavior of operator, useful to find eigenvalues + eigenvectors:

$$\hat{a}\,|v_n\rangle = \lambda_n |v_n\rangle$$

eigenvalue (λ_n), eigenvector ($|v_n\rangle$)

same data as for a physical observable!

Given observable Θ, define operator $\hat{\Theta}$ by:

$$\hat{\Theta}\,|\lambda_n\rangle = \lambda_n\,|\lambda_n\rangle.$$

In λ_n basis Θ_{mn} is diagonal: $\begin{pmatrix} \lambda_1 & & & \\ & \lambda_2 & & \\ & & \ddots & \\ & & & \lambda_n \end{pmatrix}$

Special, since $|\lambda_n\rangle$ orthogonal, λ_n real

$\Updownarrow$ MATH THEOREM

true if and only if $\hat{a}$ is HERMITIAN

$$a^*_{nm} = a_{mn}$$

matrix elements are a Hermitian matrix

$$\hat{a}^\dagger = \hat{a}$$

where ADJOINT $\hat{a}^\dagger$ defined by

$$\langle\psi_1|\hat{a}^\dagger|\psi_2\rangle = \langle\psi_2|\hat{a}|\psi_1\rangle^*$$

Fun fact: inner product $\langle\Psi|\cdot\hat{\Theta}|\Psi\rangle$ gives the expectation value of Θ in state $|\Psi\rangle$