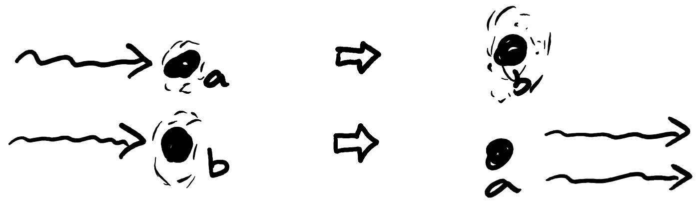
LAST TIME:

Transition rate for

ABSORPTION

STIMULATED EMISSION



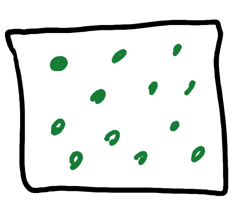
with incoherent uniform radiation:

$$R_{b \to a} = \frac{\pi}{3 \varepsilon_0 \hbar^2} \vec{\mathcal{P}}_{ab} \cdot \vec{\mathcal{P}}_{ab}^{\,*} \, \rho(\omega_{ab})$$

$\vec{\mathcal{P}}_{ab}$: $\langle \psi_a | \sum q_n \vec{x}_n | \psi_b \rangle$

$\rho(\omega_{ab})$: energy density at freq. ω_{ab}

Einstein: we can use these to figure out rate of SPONTANEOUS EMISSION.



Consider box of atoms with 2 available states $|\psi_a\rangle$, $|\psi_b\rangle$

Suppose N_a in state $|\psi_a\rangle$

N_b in state $|\psi_b\rangle$

Assume: spontaneous emission rate A

(fraction of b atoms/time making $b \to a$ transition spontaneously)

absorption rate $B_{ab} \cdot \rho(\omega_0)$

stimulated emission rate $B_{ba} \cdot \rho(\omega_0)$

Want to find A in terms of B_{ab}, B_{ba}.

Have: $$\frac{dN_b}{dt} = -N_b \cdot A - N_b \cdot B_{ba} \cdot \rho(\omega_0) + N_a \cdot B_{ab} \cdot \rho(\omega_0) \quad *$$

Assume thermal equilibrium, so N_a, N_b constant

$$* \Rightarrow \rho(\omega_0) = \frac{A}{\frac{N_a}{N_b} B_{ab} - B_{ba}}$$

Basic stat. mech: $N_a \propto e^{-E_a/kT}$ $\quad N_b \propto e^{-E_b/kT}$

$$\therefore \rho(\omega_0) = \frac{A}{e^{\hbar\omega_0/kT} B_{ab} - B_{ba}}$$

But $\rho(\omega_0)$ should be given by Planck blackbody formula

$$\rho(\omega) = \frac{\hbar}{\pi^2 c^3} \frac{\omega^3}{e^{\hbar\omega/kT} - 1}$$

$$\therefore B_{ab} = B_{ba} \quad A = \frac{\omega_0^3 \hbar}{\pi^2 c^3} B_{ba} \Rightarrow \boxed{A = \frac{\omega_0^3 \left|\vec{\wp}_{ab}\right|^2}{3\pi \varepsilon_0 \hbar c^3}}$$

SPONTANEOUS EMISSION RATE

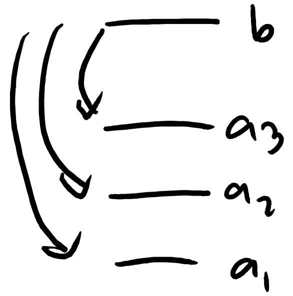
Spontaneous emission rate A for $b \to a$ means

$$\frac{dN_b}{dt} = -A \cdot N_b$$ due to $b \to a$ transitions

$$\Rightarrow N_b(t) = e^{-At} N_b(0)$$ if this is the only way to decay

$\tau = \frac{1}{A}$ is the LIFETIME of state $|\psi_b\rangle$

Generally: many different DECAY MODES



Net transition rate: $b \to$ anything $= A_1 + A_2 + \ldots$

Net lifetime $$\tau = \frac{1}{A_1 + A_2 + \ldots}$$

Which transitions are possible? Must have $E_a < E_b$, but also

$$\langle \psi_b | p^i | \psi_a \rangle \neq 0$$ for $i = x, y$ or z

H atom: $$e \langle \psi_b | x^i | \psi_a \rangle \neq 0$$ for dipole transition.

Have $\langle n' \ell' m' | \vec{x} | n \ell m \rangle = 0$ unless $\Delta \ell = \pm 1$, $\Delta m = \pm 1, 0$

SELECTION RULES.