LAST TIME: transition prob. for atom in EM field

Hamiltonian for charged particle in EM field:

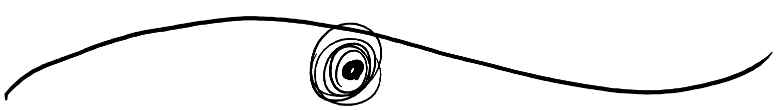
$$H = \frac{p^2}{2m} - \frac{q}{m}\vec{p}\cdot\vec{A}(\vec{x},t) + \frac{q^2}{2m}\vec{A}^2(x,t) + q\,\phi(\vec{x},t)$$

Choose $\vec{A}, \phi$ so: $\vec{\nabla}\times\vec{A} = \vec{B}$, $-\vec{\nabla}\phi - \frac{\partial\vec{A}}{\partial t} = \vec{E}$

Have: $\vec{E} = \vec{E}_0\cos(\vec{k}\cdot\vec{x} - \omega t)$ $\quad \vec{B} = \vec{B}_0\cos(\vec{k}\cdot\vec{x} - \omega t)$

Can pick: $\phi = 0$, $\vec{A} = -\frac{1}{\omega}\vec{E}_0\sin(\vec{k}\cdot\vec{x} - \omega t)$

for light:



$\lambda \sim 500\,nm \gg$ atom size

$\therefore \vec{k}\cdot\vec{x}$ small $\quad |k| = \frac{2\pi}{\lambda}$

$$\phi = 0 \qquad \vec{A} \approx \frac{1}{\omega}\vec{E}_0\sin(\omega t)$$

Simplify via gauge transformation $\phi \to \phi - \partial_t\lambda$, $\vec{A} \to \vec{A} - \vec{\nabla}\lambda$

with $\lambda = \frac{\vec{E}_0\cdot\vec{x}}{\omega}\sin(\omega t)$.

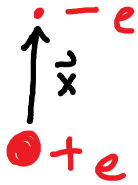
Get: $\phi_{new} = -\vec{E}_0\cdot\vec{x}\cos(\omega t)$ $\qquad \vec{A}_{new} \approx 0$

+ small terms we ignored.

Then: $H_{EM} \approx e\,\vec{E}_0 \cdot \vec{x} \cos(\omega t)$

$$= -\vec{E}_0 \cdot \vec{p} \cos(\omega t)$$

DIPOLE APPROXIMATION.



electric dipole moment operator for atom: $\vec{p} = -e\vec{x}$

General molecule: same H with $\vec{p} = \sum_i q_i \vec{x}_i$

Use this in our transition rate formula for sinusoidal time dependence:

$$P_{a\to b} = \frac{E_0^2 \left| p_{ab} \cdot \hat{E}_0 \right|^2}{\hbar^2} \frac{\sin^2((\omega-\omega_0)t/2)}{(\omega-\omega_0)^2}$$

Proportional to energy density: $u = \frac{\varepsilon_0}{2} E_0^2$

matrix element of dipole moment operator in $\vec{E}_0$ direction

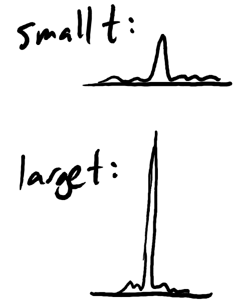
Realistic application: always have some combination of frequencies:

Define $\rho(\omega)d\omega$ = energy density in range $[\omega, \omega + d\omega]$

Assuming incoherent radiation (no phase correlations)

$$P_{b\to a} = \frac{2}{\epsilon_0 \hbar^2} \left| \vec{\mathcal{P}}_{ab} \cdot \hat{E}_0 \right|^2 \int_0^\infty d\omega \, \rho(\omega) \left\{ \frac{\sin^2((\omega_0-\omega)t/2)}{(\omega_0-\omega)^2} \right\}$$

highly peaked at $\omega=\omega_0$ at large times



$$\text{Integral} \approx \rho(\omega_0) \cdot \frac{t}{2} \cdot \pi$$

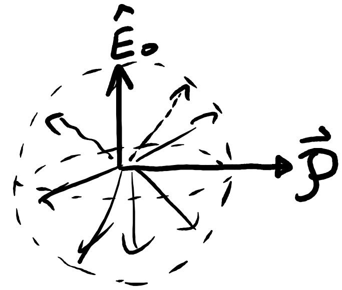
$$P_{b\to a}(t) \approx \frac{\pi \left| \mathcal{P}_{ab} \cdot \hat{E}_0 \right|}{\epsilon_0 \hbar^2} \rho(\omega_0) \cdot t$$

prob. increases linearly w. time.

$\Rightarrow$ Constant transition rate

$$R_{b\to a} = \frac{\pi}{\epsilon_0 \hbar^2} \left| \vec{\mathcal{P}}_{ab} \cdot \hat{E}_0 \right|^2 \rho(\omega_0)$$

Uniform bath of radiation: average over directions + polarizations avg of $\left|\vec{p}_{ab} \cdot \hat{E}_0\right|^2$



$$\frac{1}{4\pi}\int d\Omega \, \left|\vec{p}\cdot\hat{n}\right|^2$$

$$= \frac{1}{4\pi}\int d\phi \sin\theta \, d\theta \, \left|\vec{p}\cdot\vec{p}\right| \cos^2\theta$$

$$= \frac{1}{3}\,\vec{p}\cdot\vec{p}^{\,*}$$

FINAL TRANSITION RATE (dipole approximation)

$$\boxed{R_{b\to a} = \frac{\pi}{3\varepsilon_0 \hbar^2}\,\vec{p}_{ab}\cdot\vec{p}_{ab}^{\,*}\,\rho(\omega_{ab})}$$

$\langle \psi_a | \sum q_n \vec{x}_n | \psi_b \rangle$

energy density at freq. ω_{ab}

Valid for any atom/molecule, for ABSORPTION or STIMULATED EMISSION.