LAST TIME: Time dependent perturbation theory

$$H = H_0 + H'(t)$$

H_0: time indep

$H'(t)$: small time-dep. perturbation

$$|\psi(t)\rangle = \sum_n c_n(t)\, e^{-\frac{iE_n t}{\hbar}} |\psi_n\rangle$$

$|\psi_n\rangle$: H_0 eigenstates.

Found $c_m(t) \approx c_m(0) - \frac{i}{\hbar} \sum_n \left[\int_0^t dt'\, e^{i\omega_{mn} t'} H'_{mn}(t') \right] c_n(0) + \ldots$

$$\omega_{mn} = \frac{E_m - E_n}{\hbar} \qquad H'_{mn} = \langle m | H'(t) | n \rangle$$

TRANSITION PROBABILITIES:

if $|\psi(0)\rangle = |\psi_a\rangle$

Probability of finding $|\psi_b\rangle$ at time t:

$$P_b(t) = |c_b(t)|^2 \approx \frac{1}{\hbar^2} \left| \int_0^t dt_1\, e^{i\omega_{ba} t_1} H'_{ba}(t_1) \right|^2$$

Total probability of making a transition:

$$P_{tot} = \sum_{b \neq a} |c_b(t)|^2$$

e.g. sinusoidal perturbations

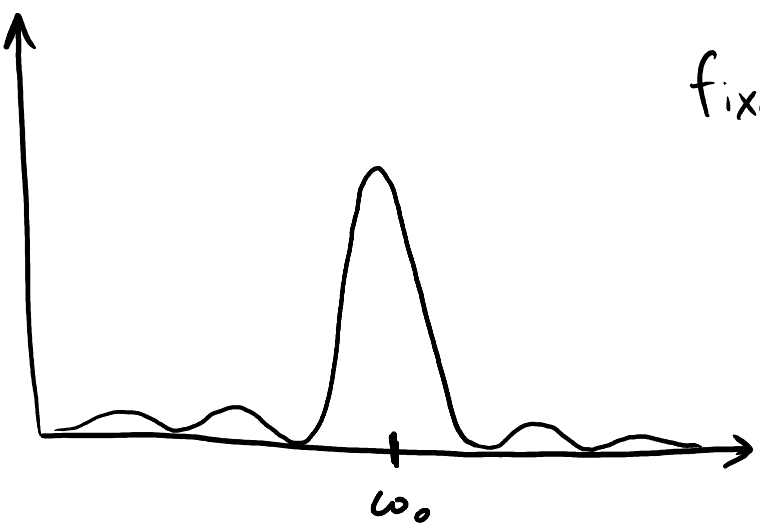
$$H'(t) = V\cos(\omega t)$$

$$P_b(t) = \frac{1}{\hbar^2}\left|\int_0^t dt'\, e^{i\omega_{ba}t'}\cos(\omega t')\,V_{ba}\right|^2$$

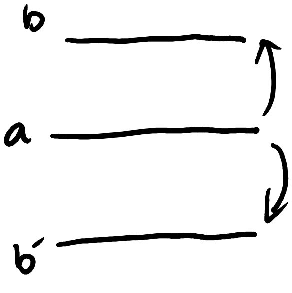
$$= \frac{|V_{ba}|^2}{\hbar^2}\left|\frac{e^{i(\omega_{ba}+\omega)t}-1}{2(\omega_{ba}+\omega)} + \frac{e^{i(\omega_{ba}-\omega)t}-1}{2(\omega_{ba}-\omega)}\right|^2$$

Most significant near $\omega \approx \omega_{ba} = \frac{|E_b - E_a|}{\hbar} \equiv \omega_0$

Then $P_b(t) \approx \frac{|V_{ba}|^2}{\hbar^2}\,\frac{\sin^2\left(\frac{(\omega-\omega_0)t}{2}\right)}{(\omega-\omega_0)^2}$



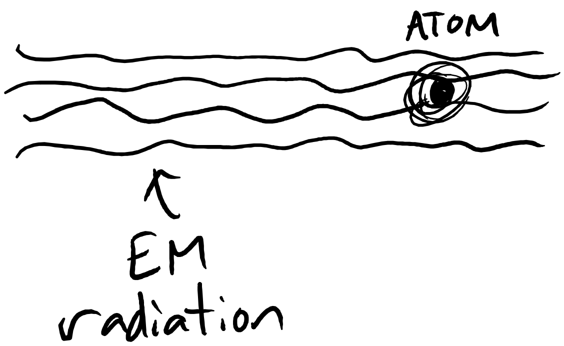
fixed t: transition probability highly peaked when driving frequency matches $\frac{|E_a - E_b|}{\hbar}$



Can stimulate transitions up (system absorbs energy from external perturbation) or down (system) emits energy.

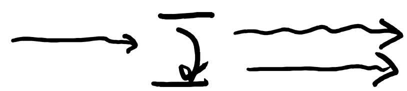
$P_b(t)$ looks sinusoidal with t, but this is an artifact of assuming a single ω.

Application: atomic/molecular transitions.



transition $\uparrow$: photon absorption

transition $\downarrow$: stimulated emission of photon



Also: can have spontaneous emission

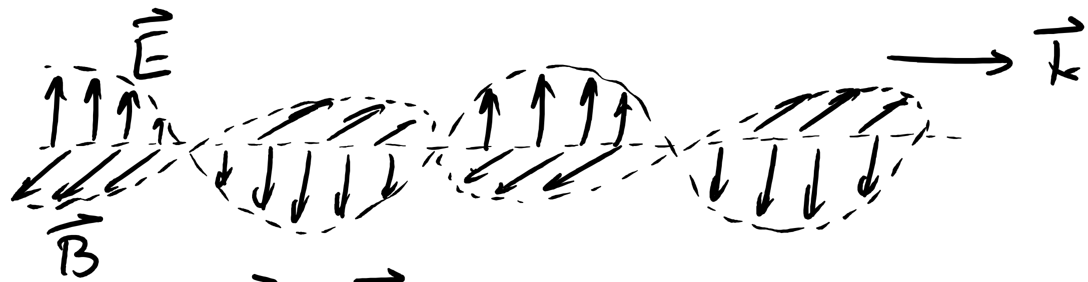
Need Hamiltonian for charged particle in EM field.

Recall: can write
$$\vec{B} = \vec{\nabla} \times \vec{A}$$
$$\vec{E} = -\frac{\partial \vec{A}}{\partial t} - \vec{\nabla}\phi$$

"Gauge transformation" $\phi \to \phi - \partial_t \lambda$ $\quad \vec{A} \to \vec{A} - \vec{\nabla}\lambda$ doesn't change $\vec{E}, \vec{B}$. Useful to pick $\vec{\nabla}\cdot\vec{A} = 0$ "Coulomb Gauge". Then:

$$H = \frac{p^2}{2m} - \frac{q}{m}\vec{p}\cdot\vec{A}(\vec{x},t) + \frac{q^2}{2m}\vec{A}^2(x,t) + q\,\phi(\vec{x},t)$$

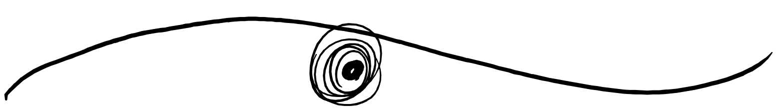
Monochromatic EM wave:



$$\vec{E} = \vec{E}_0 \cos(\vec{k}\cdot\vec{x} - \omega t) \qquad |\vec{B}_0| = \frac{1}{c}|\vec{E}_0|$$
$$\vec{B} = \vec{B}_0 \cos(\vec{k}\cdot\vec{x} - \omega t)$$

Can choose $\phi = 0 \quad \vec{A} = -\frac{1}{\omega}\vec{E}_0 \sin(\vec{k}\cdot\vec{x} - \omega t)$

for light:



$\lambda \sim 500\,\text{nm} >>$ atom size

$\therefore \vec{k}\cdot\vec{x}$ small $\quad |k| = \frac{2\pi}{\lambda}$

$$\phi = 0 \qquad \vec{A} \approx \frac{1}{\omega}\vec{E}_0 \sin(\omega t)$$

Simplify via gauge transformation with $\lambda = \frac{\vec{E}_0\cdot\vec{x}}{\omega}\sin(\omega t)$

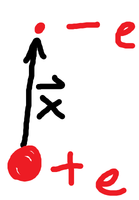
$$\phi_{new} = -\vec{E}_0\cdot\vec{x}\cos(\omega t) \qquad \vec{A}_{new} \approx 0$$

+ small terms we ignored.

Then: $H_{EM} \approx e\,\vec{E}_0\cdot\vec{x}\cos(\omega t)$

$\qquad = -\vec{E}_0\cdot\vec{p}\cos(\omega t)$

DIPOLE APPROXIMATION



electric dipole moment operator for atom: $\vec{p} = -e\vec{x}$