LAST TIME:

① Physical states $\longrightarrow$ Vectors in a Hilbert space

② For each observable $\mathcal{O}$, have orthonormal basis $|\lambda_n\rangle$ of states with definite value λ_n for $\mathcal{O}$

qubit example: dimension 2 Hilbert space

$$|\psi\rangle = z_1 |\uparrow\rangle + z_2 |\downarrow\rangle$$

basis elements: eigenstates of S_z with $\lambda = \pm \frac{\hbar}{2}$

equivalent to

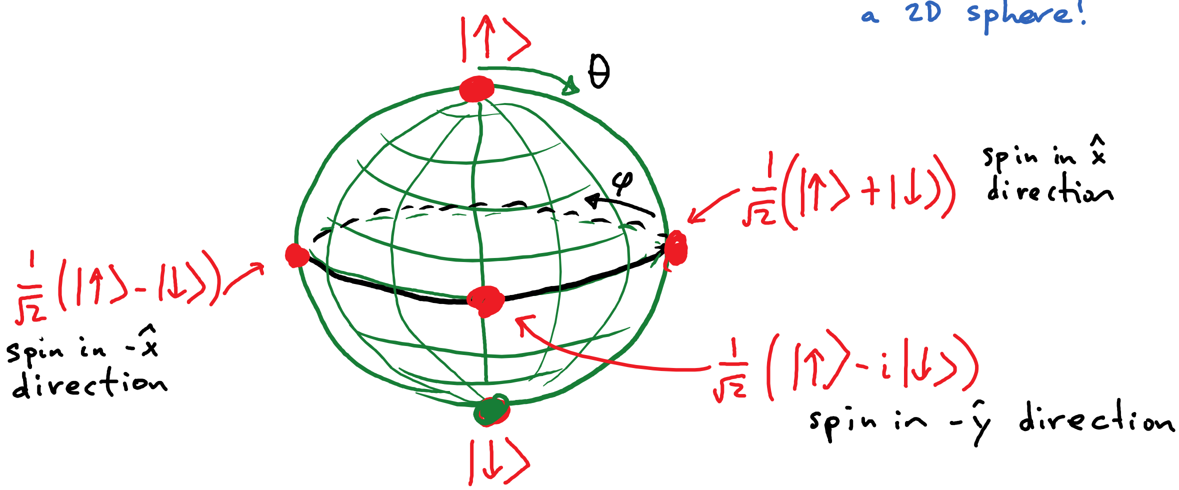
$$|\psi\rangle = \cos\left(\frac{\theta}{2}\right)|\uparrow\rangle + e^{i\varphi}\sin\left(\frac{\theta}{2}\right)|\downarrow\rangle$$

for some $0 \leq \theta \leq \pi$, $0 \leq \varphi \leq 2\pi$

same parameters as for a 2D sphere!

States of a QUBIT: (e.g. spin states of electron)

opposite points on sphere correspond to orthogonal states

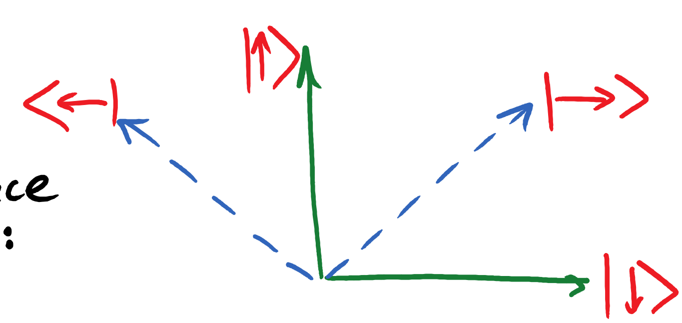


different observables $\longleftrightarrow$ different bases

e.g. S_x: $S_x = \frac{\hbar}{2}$ $\quad |\rightarrow\rangle = \frac{1}{\sqrt{2}}|\uparrow\rangle + \frac{1}{\sqrt{2}}|\downarrow\rangle$

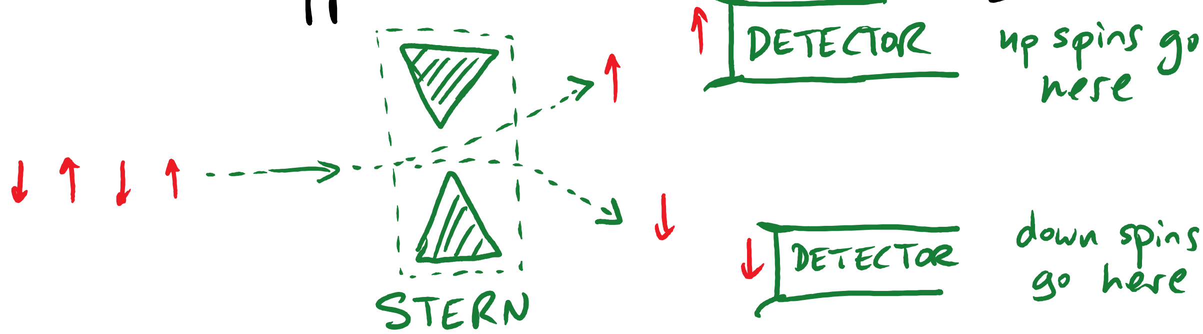
$S_x = -\frac{\hbar}{2}$ $\quad |\leftarrow\rangle = \frac{1}{\sqrt{2}}|\uparrow\rangle - \frac{1}{\sqrt{2}}|\downarrow\rangle$

Hilbert space picture:



Can express any state *either* in terms of $|\uparrow\rangle, |\downarrow\rangle$ *or* in terms of $|\leftarrow\rangle, |\rightarrow\rangle$

PHYSICS: what happens if we measure S_z?



STERN -GERLACH APPARATUS

What about states with spin in $\hat{x}$ direction $\equiv |\rightarrow\rangle$

Write: $|\rightarrow\rangle = \frac{1}{\sqrt{2}}|\uparrow\rangle + \frac{1}{\sqrt{2}}|\downarrow\rangle$ superposition of S_z eigenstates

Will behave as $|\uparrow\rangle$ or $|\downarrow\rangle$ in measurement of S_z. Probabilities determined by coefficients in superposition: $\left|\frac{1}{\sqrt{2}}\right|^2 = \frac{1}{2}$ for both.

Q: give two examples of systems with a 4d Hilbert space

A: simple example: spin states of two electrons!

basis elements: $|\uparrow\uparrow\rangle, |\uparrow\downarrow\rangle, |\downarrow\uparrow\rangle, |\downarrow\downarrow\rangle$

general state: $z_1|\uparrow\uparrow\rangle + z_2|\uparrow\downarrow\rangle + z_3|\downarrow\uparrow\rangle + z_4|\downarrow\downarrow\rangle$

or: spin states of spin $\frac{3}{2}$ system

generally spin j has dimension $2j+1$ dimensional Hilbert space, basis $|j\ m\rangle$

Q: How many parameters do we need to describe the most general state of N qubits?

A: $2 \times 2^N - 2$: Have 2^N basis elements $\therefore 2^N$ complex $= 2 \cdot 2^N$ real coefficients

- can eliminate 2 by normalization + phase rotation