LAST TIME: "Fine structure" of atomic hydrogen spectrum:

- Spin orbit coupling:

$$H_{s.o.} = \frac{g e^2}{16\pi\varepsilon_0 m^2 c^2} \frac{1}{r^3} \vec{S}\cdot\vec{L}$$

Matrices for degenerate Pert. theory are diagonal in H_0, L^2, J^2, J_z basis $|n\ \ell\ J\ M\rangle$:

Get

$$\delta E_{s.o.} = \frac{g e^2 \hbar^2}{32\pi^2\varepsilon_0 m^2 c^2} \frac{\left(J(J+1) - \ell(\ell+1) - \frac{3}{4}\right)}{\ell\left(\ell+\frac{1}{2}\right)(\ell+1)n^3 a^3}$$

Similar size correction from relativistic correction to electron energy:

from $p = \gamma m v$

$E = \gamma m c^2$

Recall total energy: $E^2 = p^2c^2 + m^2c^4$

kinetic energy: $E_k = \sqrt{p^2c^2 + m^2c^4} - mc^2$

$$\approx \frac{p^2}{2m} - \frac{p^4}{8m^3c^2} + \dots$$

perturbation

represent $p_i = \frac{\hbar}{i}\frac{\partial}{\partial x_i}$ acting on wavefunction

$$\langle n\, l\, J\, M | -\frac{p^4}{8m^3c^2} | n\, l\, J\, M\rangle = -\frac{E_n^2}{2mc^2}\left(\frac{4n}{l+\frac{1}{2}} - 3\right)$$

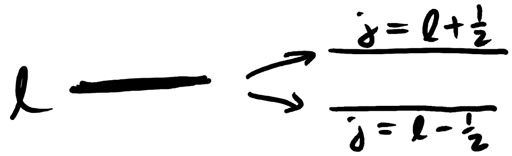
TOTAL $$\boxed{E_n + \delta E^{s.o.} + \delta E^{rel} = -\frac{13.6\,eV}{n^2}\left(1 + \frac{\alpha^2}{n^2}\left(\frac{n}{j+\frac{1}{2}} - \frac{3}{4}\right)\right)}$$

Fractional correction to leading part $\alpha = \frac{e^2}{4\pi\epsilon_0\hbar c} \approx \frac{1}{137}$

α = "fine structure constant"

- measures strength of electromagnetic interactions
- physics would be extremely hard if $\alpha \approx 1$

Corrections break some degeneracy



$\therefore$ level n splits to n levels

Some degeneracy remains since still have rotation symmetry (J_i acting on state preserves energy)

Can take into account spin-orbit effects & relativistic effects exactly using "Dirac equation"

Full result:

$$E_{nj} = mc^2\left\{\left[1 + \left(\frac{\alpha}{n - (j+\frac{1}{2}) + \sqrt{(j+\frac{1}{2})^2 - \alpha^2}}\right)^2\right]^{-\frac{1}{2}} - 1\right\}$$

still not exact: extra effects from proton spin and from quantizing electromagnetic field.

Hyperfine splitting:

proton also has spin $\frac{1}{2}$, magnetic moment $\vec{\mu}_p = \frac{g_p e}{2m_p}\vec{S}_p$

$g_p \approx 5.59$

$\rightarrow$ produces magnetic field

$$\vec{B}_p = \frac{\mu_0}{4\pi r^3}\left[3(\vec{\mu}\cdot\hat{r})\hat{r} - \vec{\mu}\right] + \frac{2\mu_0}{3}\vec{\mu}\,\delta^3(\vec{r})$$

Electron magnetic moment interacts as: $H_{h.f.} = -\mu_e \cdot \vec{B}_p$

Tiny effect, but important for ground states:

$$|g_i\rangle = |1\ 0\ 0\rangle \otimes |s_z^e\rangle \otimes |s_z^p\rangle \leftarrow 4 \text{ states}$$

For these states,

$$\langle g_i | H_1 | g_j \rangle = \frac{\mu_0 g e^2}{3\pi m_p m_e a^3} \langle g_i | \vec{S}_p \cdot \vec{S}_e | g_j \rangle$$

Use same trick: $\vec{S}_p \cdot \vec{S}_e = \frac{1}{2}\left(\vec{S}_{tot}^{\,2} - \vec{S}_p^{\,2} - \vec{S}_e^{\,2}\right)$

$\vec{S}^{tot} = \vec{S}_p + \vec{S}_e$ $\quad\therefore$ use S_{tot}^2, S_z^{tot} basis

spin $\frac{1}{2}$ $\otimes$ spin $\frac{1}{2}$

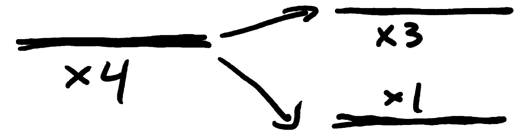
$\rightarrow$ spin 1 $\quad |\uparrow\rangle\otimes|\uparrow\rangle,\ \frac{1}{\sqrt{2}}\left(|\uparrow\rangle\otimes|\downarrow\rangle + |\downarrow\rangle\otimes|\uparrow\rangle\right),\ |\downarrow\rangle\otimes|\downarrow\rangle$

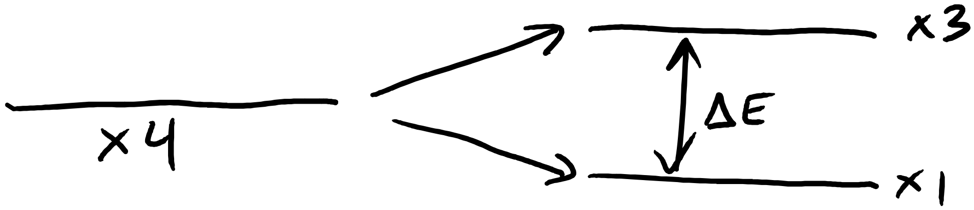
"TRIPLET"

$\rightarrow$ spin 0 $\quad \frac{1}{\sqrt{2}}\left(|\uparrow\rangle\otimes|\downarrow\rangle - |\downarrow\rangle\otimes|\uparrow\rangle\right)$

"SINGLET"

$$\delta E_{h.f.} = \frac{4g\hbar^4}{3m_p m_e^2 c^2 a^4}\begin{cases} \frac{1}{4} & \text{triplet} \\ -\frac{3}{4} & \text{singlet} \end{cases}$$





$$\Delta E = 5.88 \times 10^{-6}\,\text{eV} = |E_0| \cdot \frac{8}{3} \cdot g_p \cdot \alpha^2 \cdot \left(\frac{m_e}{m_p}\right)$$

$\longrightarrow$ triplet $\rightarrow$ singlet transition gives radiation with freq. $\nu = \frac{\Delta E}{h} = 1420\,\text{MHz}$

wavelength $\lambda = \frac{c}{\nu} = 21\,\text{cm}$

★ Radio wave signal for atomic hydrogen ★