LAST TIME: Rotations in QM

$R(\hat{n},\theta)$ acts on Hilbert space as unitary operator: $\hat{\tau}(R(\hat{n},\theta))$

Infinitesimal: $\hat{\tau}(R(\hat{n},\varepsilon)) = \mathbb{1} - \frac{i}{\hbar}\varepsilon\, \hat{n}\cdot\vec{J}$

Finite rotation: $\hat{\tau}(R(\hat{n},\theta)) = e^{-\frac{i}{\hbar}\theta\,\hat{n}\cdot\vec{J}}$

$\vec{J} = (J_x, J_y, J_z)$: angular momentum operators

Hilbert space of a quantum system divides into subspaces with definite total angular momentum:

Can label states in a subspace by $|J\,M\rangle$ where:

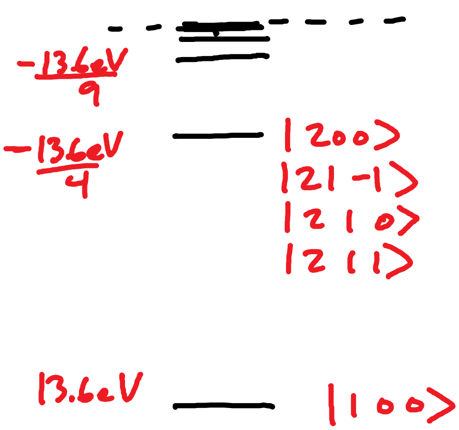
$$J^2 |J\,M\rangle = \hbar^2 J(J+1)\,|J\,M\rangle$$

$$J_z |J\,M\rangle = \hbar M\,|J\,M\rangle$$

$$J_+ |J\,M\rangle = \hbar\sqrt{J(J+1) - M(M+1)}\,|J\,M+1\rangle$$

$$J_- |J\,M\rangle = \hbar\sqrt{J(J+1) - M(M-1)}\,|J\,M-1\rangle$$

e.g.



H atom (ignoring spin)

eigenstates $|n\,\ell\,m\rangle$

energy $E_n = \frac{-13.6\text{eV}}{n^2}$

usually call angular momentum operators acting on position state of electron in H atom $\hat{L}_i$, $\hat{L}^2$

determines L^2

$|n\,\ell\,m\rangle$

determines L_z

Real H atom: electron has spin states; rotations affect these.

One basis: $|n\ \ell\ m\rangle \otimes |s_z\rangle$ 2-part system.

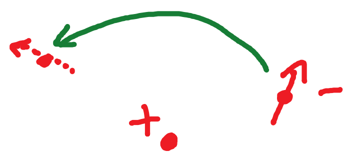
position state

spin state.

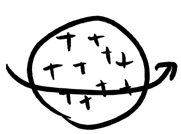
Rotation operators are $\vec{J} = \vec{L} + \vec{S}$

acts on position part.

acts on spin part



Electron has magnetic moment (acts as magnet) from spin + charge.



For classical ball of charge: $\vec{\mu} = \frac{q}{2m}\vec{L}$

for electron: $\vec{\mu} = -\frac{g \cdot q}{2m}\vec{L}$

$g = 2.00231930436 1$

most accurately verified number in science

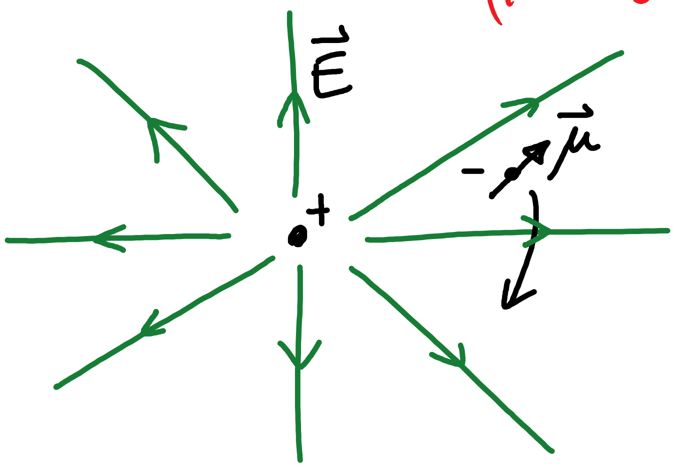
In magnetic field: $\vec{\mu}$ wants to align

have extra term $H = -\vec{\mu}\cdot\vec{B}$

In H atom: no $\vec{B}$ but have interaction between *moving* dipole and electric field of proton

Full Hamiltonian includes extra interaction

$$H_{s.o.} = \left(\frac{ge^2}{16\pi\varepsilon_0}\frac{1}{m^2c^2}\right)\cdot\frac{1}{r^3}\cdot\vec{S}\cdot\vec{L}$$



from moving magnetic dipole in electric field.

To find effects of $H_{s.o.}$, use degenerate perturbation theory: want basis where $H_{s.o.}$ is diagonal on subspace of states with same n.