LAST TIME: Perturbation theory.

Want to find energy eigenstates of $H = H_0 + \lambda H_1$ where H_0 has eigenstates $|\psi_n^0\rangle$ with energy E_n^0.

Write: $E_n(\lambda) = E_n^0 + \lambda E_n^1 + \lambda^2 E_n^2 + \ldots$

$$|\psi_n(\lambda)\rangle = |\psi_n^0\rangle + \lambda |\psi_n^1\rangle + \lambda^2 |\psi_n^2\rangle + \ldots$$

Solve $(H_0 + \lambda H_1)|\psi_n(\lambda)\rangle = E_n(\lambda)|\psi_n(\lambda)\rangle$

together with $\langle\psi_n(\lambda)|\psi_n(\lambda)\rangle = 1$

OR $\langle\psi_n^0|\psi_n(\lambda)\rangle = 1$

need to normalize state at the end if we do this

RESULTS:

$$\boxed{E_n^1 = \langle\psi_n^0|H_1|\psi_n^0\rangle \qquad |\psi_n^1\rangle = \sum_{m\neq n} |\psi_m^0\rangle \frac{\langle\psi_m^0|H_1|\psi_n^0\rangle}{E_n^0 - E_m^0}}$$

problem if $E_m = E_n$!

Next order: $H_0|\psi_n^2\rangle + H_1|\psi_n^1\rangle = E_n^0|\psi_n^2\rangle + E_n^1|\psi_n^1\rangle + E_n^2|\psi_n^0\rangle$

Take inner product with $|\psi_n^0\rangle$ to get:

$$E_n^2 = \langle\psi_n^0|H_1|\psi_n^1\rangle = \sum_{m\neq n} \frac{|\langle\psi_m^0|H_1|\psi_n^0\rangle|^2}{E_n^0 - E_m^0}$$