BASICS OF QUANTUM COMPUTING

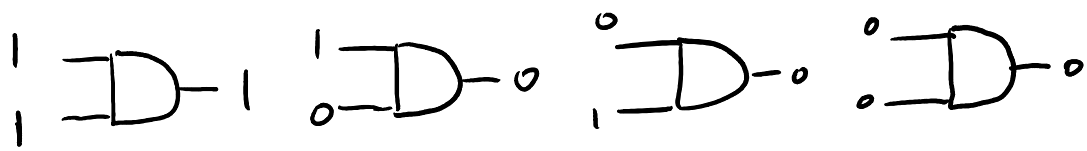
Computer: start with BITS in some state

e.g. 1 0 1 1 0 0 1 0 1 1 1 0 0 1

apply an ALGORITHM via basic GATES

e.g.

AND gate:



read result from final state of bits

Quantum computer: use QUBITS:

Basis states $|\uparrow\downarrow\downarrow\uparrow\downarrow\ldots\uparrow\downarrow\rangle$ like ordinary bits.

BUT general state is a quantum superposition:

$$\sum \psi_{s_1 \ldots s_n} |s_1 s_2 \ldots s_n\rangle \qquad 2^n \text{ coefficients}$$

classical computer: each extra bit: one more bit of information

quantum computer: each extra bit: doubles amount of information

Quantum gates: unitary transforms on a few qubits

SINGLE QUBIT GATES:

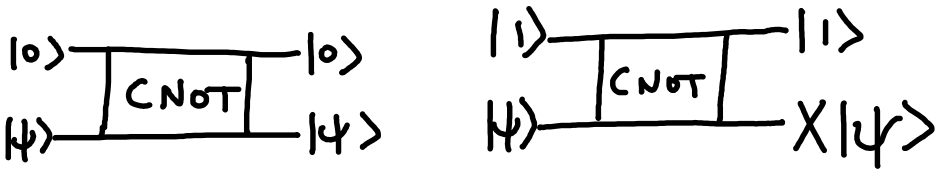
e.g. $|\uparrow\rangle - \boxed{X} - |\downarrow\rangle \qquad |\downarrow\rangle - \boxed{X} - |\uparrow\rangle \qquad \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

Hadamard:

$|\uparrow\rangle \to \frac{1}{\sqrt{2}}(|\uparrow\rangle + |\downarrow\rangle) \qquad |\downarrow\rangle \to \frac{1}{\sqrt{2}}(|\uparrow\rangle - |\downarrow\rangle) \qquad \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$

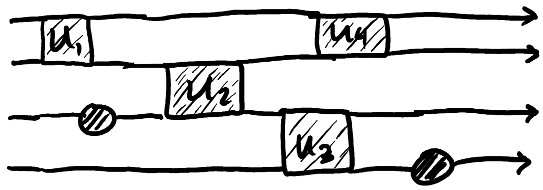
2 qubit gate: 

e.g. CNOT = "controlled NOT"



★ Any multi-qubit gate can be built from CNOT + single qubit gates

Quantum circuit



Quantum computation: prepare initial state

apply unitary gates (quantum circuit)

Make a measurement (perhaps repeated)

(e.g. 1st qubit up → YES to some yes/no question)
down → NO

Quantum computers believed to be better than classical computers for e.g. factoring large numbers

certain search algorithms

simulating quantum systems.

★ Hard to put into practice. Currently ~50 qubits but with errors ★

Example of how quantum computers are powerful:

Let $\vec{f}(\vec{n})$ be a function from n-digit binary numbers to N-digit binary numbers.

Suppose we build a quantum computer that "computes" this via:

$$\text{INPUT} \quad |\overset{\vec{n}}{10110\ldots01}\rangle\,|\overset{\text{OUTPUT}}{00\ldots0}\rangle$$

$$\downarrow$$

$$|\underbrace{10110\ldots01}_{\vec{n}}\rangle\,|\underbrace{1110\ldots101}_{f(\vec{n})}\rangle$$

If $\quad |\vec{n}\rangle|\vec{0}\rangle \rightarrow |\vec{n}\rangle|\vec{f}(\vec{n})\rangle$ then:

$$\frac{1}{\sqrt{2^N}}\sum_{\vec{n}}|\vec{n}\rangle|\vec{0}\rangle \longrightarrow \frac{1}{\sqrt{2^N}}\sum_{\vec{n}}|\vec{n}\rangle|f(\vec{n})\rangle$$

Putting in this input state gives an output that knows about all 2^N values of the function.

Can't extract all this info, but can use this to do v. complicated calculations.