The Fundamentals of Quantum Mechanics

① Configuration of system at time t specified by a STATE $|\psi(t)\rangle \longrightarrow$ VECTOR in a HILBERT SPACE

any linear combination (with complex coefficients) of states is a state

$|\psi\rangle$ and $z \cdot |\psi\rangle$ represent same state

there is an INNER PRODUCT

like a dot product

$(|A\rangle, |B\rangle) \rightarrow \langle A|B\rangle = \langle B|A\rangle^*$

$\langle A|A\rangle \geq 0$ equality if and only if $|A\rangle = 0$

↳ usually: assume $\langle\psi|\psi\rangle = 1$ and $|\psi\rangle \equiv e^{i\varphi}|\psi\rangle$

② Physical quantities do not have definite values in a general state

- For a given observable $\mathcal{O}$ (e.g. energy, position) there are special states that do have specific values: EIGENSTATES of $\mathcal{O}$

★ these span the full set of states ★

→ Can form orthonormal basis from $\mathcal{O}$ eigenstates

can write any state uniquely as $|\psi\rangle = \sum_n z_n |\lambda_n\rangle$

eigenstate: labeled by value of $\mathcal{O}$

orthonormal $\Rightarrow \sum_n |z_n|^2 = 1$ if $\langle\psi|\psi\rangle = 1$

Interpretation: $|z_n|^2$ is the probability of finding the result λ_n if we measure $\mathcal{O}$ in state $|\psi\rangle$.

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simplest example: QUBIT = dimension 2 Hilbert space

\# of independent basis vectors

e.g. spin states of an electron, proton or neutron.