A GROUP is a set G with a "multiplication rule" $\cdot$ between elements that gives an element $g_1 \cdot g_2$ of G for any ordered pair of elements (g_1, g_2), such that:

- ★ there is an IDENTITY ELEMENT 1 in G satisfying $1 \cdot g = g \cdot 1 = g$ for any g in G.
- ★ each element g has an INVERSE element g^{-1} satisfying $g \cdot g^{-1} = g^{-1} \cdot g = 1$
- ★ the multiplication rule is ASSOCIATIVE: for any g_1, g_2, g_3, we must have:

$$g_1 \cdot (g_2 \cdot g_3) = (g_1 \cdot g_2) \cdot g_3$$

★ it is not required that $g_1 \cdot g_2 = g_2 \cdot g_1$. If this property (called COMMUTATIVITY) holds, the group is called an ABELIAN GROUP

★ groups with a finite number of elements are called FINITE GROUPS. There are also infinite groups

---

Here are the simplest finite groups:

1 element: $G = \{1\}$ with multiplication rule $1 \cdot 1 = 1$

2 elements: $G = \{1, a\}$ with multiplication table

2nd element g_2

1st element g_1 $\cdot$	1	a
1	1	a
a	a	1

← product $g_1 \cdot g_2$

Exercises:

① There is one group with 3 elements. If we take $G = \{1, a, b\}$, what is the multiplication table?

	1	a	b
1			
a			
b			

② There are two different groups with 4 elements. Can you figure out their multiplication tables?

	1	a	b	c
1				
a				
b				
c				

	1	a	b	c
1				
a				
b				
c				

NOTE: two groups are considered the same if you can go from one to the other just by re-labeling the elements.

③ Show that the non-zero real numbers together with the usual multiplication rule form a group.

④ If G is a group, a SUBGROUP is a subset of elements which also form a group using the same multiplication rule.
What subgroups of the group in problem 3 can you think of?