Problem 1: short answers

a) What two physical effects together give the largest corrections to the energy levels of the hydrogen atom (known as the fine structure corrections) once we go beyond the simple Coulomb potential approximation? What are the corresponding operators that must be added to the Hamiltonian to take into account these effects (don't worry about the constants in front of the operators)? (*2 points*)

b) If we wish to determine the absorption rate for allowed $|100\rangle$ to $|311\rangle$ transitions in atomic hydrogen, what matrix elements do we need to calculate? What other property of the system do we need to know to determine the rate? (*2 points*)

c) What are forbidden transitions? (*1 point*)

d) For a particle in a one-dimensional potential $V(x)$, we wish to determine an upper bound on the ground state energy using the variational method with the trial wavefunction

$$\psi(x) = \frac{A}{b^2 + x^2}$$

Briefly describe the steps we need to take. (*3 points*)

Problem 2

a) A particle of charge q and spin 1 sits in a magnetic field $\vec{B} = B\hat{z}$. What are the energy eigenvalues for this system (assume the gyromagnetic ratio is $g = 1$), ignoring the motion of the particle? (*1 point*)

b) If we perturb the system by adding a Hamiltonian

$$H' = \alpha(S_x^2 - S_y^2)$$

what is the shift in the ground state energy at the first nonzero order? (*4 points*)

c) What is the exact ground state energy of the perturbed system? (*1 point*)

Problem 3

a) Consider a spin 1 particle at some fixed location (and no magnetic field). If we can ignore the motion of the particle, the Hamiltonian can be taken to be simply $H = 0$. If we now perturb this system by the same perturbation as in question 2,

$$H' = \alpha(S_x^2 - S_y^2) \, ,$$

what are the energy eigenvalues at first order in perturbation theory? (*4 points*)

b) If we measure S_z in the lowest energy state of the perturbed system, what are the possible values we might obtain, and what are the probabilities of each? (*2 points*)

Problem 4

A particle is in the first excited state of a one-dimensional harmonic oscillator with frequency Ω. At time $t = 0$, a perturbation

$$H' = Ax^3$$

is turned on, and later turned off again at $t = \pi/\Omega$. After this, the energy of the particle is measured.

a) Considering only first-order effects of the perturbation, what are the possible results for the measurement of energy, and what are the probabilities for obtaining them? (*5 points*)

b) What is the condition on A so that these leading order results reliable? (*1 point*)

Problem 6

Consider two electrons confined in a three dimensional harmonic oscillator potential.

a) Write expressions for an independent basis of states for the lowest two energy levels. Also, give the energy and degeneracy for these levels. (*2 points*)

b) For the ground state(s) what is the expectation value of J^2? (*1 points*)

c) Repeat part a) in the case where we now turn on a very strong magnetic field, so that only spin down electrons are allowed. (*2 points*)

d) In this case, what is the expectation value of J^2 for the ground state(s)? (*1 point*)

Problem 7

If the time-independent operator $\hat{\mathcal{O}}$ corresponding to some observable commutes with the Hamiltonian, show that the expectation value of $\mathcal{O}$ for any state is independent of time. (*2 points*)

40 points available