Physics 402 Exam

April 19, 2006

Problem 1: Short Answer

a) What two physical effects lead to the "fine structure" of the Hydrogen atom spectrum? (*2 points*)

b) An electron in a hydrogen atom has $l = 2$, $J = 3/2$, and $M = 1/2$, where l, J, and M are the quantum numbers corresponding to L^2, J^2, and J_z respectively. What is the expectation value for a measurement of L_z in this state? (*2 points*)

c) Real atomic spectral lines always contain a range of frequencies, rather than just a single frequency. Explain one reason for this. (*2 points*)

Problem 2

Consider two (spin 1/2) electrons in an infinite one-dimensional square well of width a.

a) Assuming that the electrons do not interact with each other (but still taking into account the fact that they are fermions), what are the energies and degeneracies for the two lowest energy levels? (*2 points*)

b) Now suppose we turn on a very large magnetic field $\vec{B} = B\hat{z}$. What are the energy levels and degeneracies for the two lowest energy levels now? (*2 points*)

c) In terms of the basis $|n_1\ s_z^1\rangle \otimes |n_2\ s_z^2\rangle$, what is (are) the ground state(s) for parts a) and b)? (*2 points*)

Problem 3

a) What does the variational principle tell us? (*1 point*)

b) Consider a spin 1/2 particle governed by a Hamiltonian

$$H = \alpha(S_x + S_y)$$

where α is a constant. Using the variational method, determine the best upper bound on the ground state energy coming from a trial state

$$|\Psi>= A|S_z = \hbar/2\rangle + B|S_z = -\hbar/2\rangle$$

where A and B are real. (*7 points*)

c) What is the exact ground state energy? (*1 point*)

Problem 4

a) A particle is in the ground state of an infinite square well potential of width L at $t = -\infty$. The well is then perturbed by a delta function spike which gradually rises in the center of the potential and then gradually disappears,

$$H'(t) = A\delta(x)e^{-bt^2}$$

If we measure the energy at $t = \infty$, what are the possible values we might obtain, and what are the probabilities of obtaining them (assuming first order perturbation theory is valid). (*7 points*)

b) What feature of the result for the transition probabilities does the adiabatic theorem predict?

NOTE: The following integral may be useful (valid for α real and positive):

$$\int_{-\infty}^{\infty} e^{-\alpha t^2+\beta t}dt = \sqrt{\frac{\pi}{\alpha}}\, e^{\frac{\beta^2}{4\alpha}}$$

Problem 5

a) An electron is in the $|n_x\ n_y\ n_z\rangle = |1\ 0\ 0\rangle$ state of a three-dimensional harmonic oscillator potential with frequency ω (you could imagine that this potential is due to a special kind of proton with a harmonic oscillator potential instead of a Coulomb potential). Calculate the rate for spontaneous transitions to the ground state (due to electric dipole transitions). (*7 points*)

b) If the electron begins in the general state $|n_x\ n_y\ n_z\rangle$, to which other states are dipole transitions possible? (*1 point*)

Problem 6

Consider two non-interacting particles in the ground state of a one-dimensional harmonic oscillator with frequency ω (you can assume they are spinless bosons).

a) If somehow we turn on an interaction between the particles described by a potential

$$V = \frac{1}{2}k(x_1 - x_2)^2 ,$$

calculate the new ground state energy to first order in perturbation theory. (*6 points*)

b) For which values of k is your result from part a) reliable? (*1 point*)

c) What are the exact energy levels for this system? (*1 point*)

Total available points: 45