① SHORT ANSWER QUESTIONS: (2 pts each)

a) What does the VARIATIONAL PRINCIPLE tell us?

b) To which states $|n\ \ell\ m\rangle$ can the $|4\ 2\ 1\rangle$ state of atomic hydrogen decay via a single allowed dipole transition?

c) Taking into account electron spin, proton spin, and orbital angular momentum, what are the possible values for total angular momentum in the $n=2$, $\ell=1$ state of hydrogen, and how many independent states are there with each of the allowed values for $\vec{j}$?

② For this question, suppose we can ignore interactions between electrons and all fine structure effects.

a) What is the ground state energy of the Li^+ ion ($Z=3$ nucleus with 2 electrons)? (2 pts)

b) In terms of the basis $|n\ \ell\ m\ s_z\rangle \otimes |n'\ \ell'\ m'\ s_z'\rangle$ of states for the 2 electrons, what is the ground state? (2 pts)

c) If we measure the total angular momentum in the $\hat{z}$ direction ($J_z = L_z^1 + L_z^2 + S_z^1 + S_z^2$) what are the possible values we might obtain? (2 pts)

d) What is the energy and degeneracy for the 1st excited level? (2 pts)

** For parts a) and d) you may express the energies in terms of the ground state energy $E_0 = -13.6\,eV$ of hydrogen **

(3) Consider a time dependent hamiltonian

$$H = \frac{p^2}{2m} + \frac{1}{2} m \left(\omega^2 + \Delta(t)\right) x^2$$

where:

$$\Delta(t) = \begin{cases} \Delta \sin\left(\frac{t}{T}\pi\right) & 0 \leq t \leq T \\ 0 & \text{otherwise} \end{cases}$$

a) The particle begins in the ground state of the harmonic oscillator for $t < 0$. Assuming Δ is sufficiently small, calculate the probability that the particle will be in the state $|n\rangle$ (for each $n > 0$) after a time T, using first order time-dependent perturbation theory. (7pts)

b) For what values of Δ do you expect your result to be valid? (1 pt)

⑥ Consider a 3-dimensional harmonic oscillator, with Hamiltonian

$$H = \frac{p_x^2}{2m} + \frac{p_y^2}{2m} + \frac{p_z^2}{2m} + \frac{1}{2} m\omega^2 (x^2 + y^2 + z^2)$$

a) What are the energies and degeneracies for the two lowest energy levels? (2 pts)

b) Since the potential is spherically symmetric, it is possible to choose a basis of energy eigenstates that are also eigenstates of L_z and L^2. For the first excited level, what are the eigenstates of L_z (in terms of states built with creation and annihilation operators)? (3 pts)

c) What are the energy shifts for the first excited level if we add a perturbation $H_1 = \alpha L^2 + \beta L_z$? (2 pts)

d) (BONUS QUESTION) Before adding the perturbation, what are the possible values for (ℓ, m) in the nth excited level? (1 pt)