1. Short Answer Questions:

   a) What are the values of (l, m, s_z) in the $\{L^2, L_z, S_z\}$ basis states for an electron in the $n = 2$ state of a hydrogen atom? What are the values of (l, J, M) for the $\{L^2, J^2, J_z\}$ basis of $n = 2$ states? *(3 points)*

   b) Upon which physical quantities does the absorption rate for an allowed transition between two states $|\psi_a\rangle$ and $|\psi_b\rangle$ in an atomic system depend if the atom is in isotropic incoherent radiation? *(2 points)*

   c) The energy levels for a particle in an infinite square well potential of length a are $E_n = \frac{n^2\pi^2\hbar^2}{2ma^2}$. At time $t = 0$, the energy is measured to be $\frac{18\pi^2\hbar^2}{ma^2}$. The potential is then very gradually increased in length to a final length $3a$. If we then measure the energy again, what is its expected value? *(2 points)*

2. What is the first non-zero energy shift for the ground state of a harmonic oscillator when a perturbation $H_1 = \lambda x^3$ is added, where λ is small. How small does λ have to be for your result to be reliable? *(6 points)*

4. The wavefunction for the ground state of a harmonic oscillator is given by

$$\psi_0(x) = \langle x|0\rangle = \left(\frac{m\omega}{\pi\hbar}\right)^{\frac{1}{4}} e^{-\frac{m\omega x^2}{2\hbar}}$$

   Using the fact the the first excited state is given by $|1\rangle = a^\dagger|0\rangle$, determine the wavefunction for the first excited state (You should not need to evaluate any integrals). *(5 points)*

5. An electron is in the spin state $|S_z = \frac{\hbar}{2}\rangle$ at $t = 0$. A magnetic field

$$\vec{B}(t) = B(\hat{x}at + \hat{y}bt^2 + \hat{z}ct^3)$$

   is turned on at $t = 0$. Assuming B is small, what is the probability of measuring the particle in state $|S_z = -\frac{\hbar}{2}\rangle$ at a later time $t = T$ (assume also that T is not too large)? *(6 points)*

6. a) What are the energy eigenstates and eigenvalues for a particle of spin 1 whose time evolution is governed by a Hamiltonian $H = \alpha L_z^2$. *(1 points)*

   b) If we add a small perturbation $H_1 = \beta L_x^2$, what are the shifted energy levels to first order in perturbation theory? *(4 points)*

   c) If we increase the perturbation to $\beta = \alpha$, what are the exact energies? (Hint: there is an easy way to do this). *(1 point)*