Physics 402 Exam, April 26, 2017

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Abstract

In this exam, an attempt is made to demonstrate proficiency in the fundamentals and various applications of quantum mechanics. Conceptual understanding is displayed by suggesting answers to various multiple choice questions. Further understanding and technical proficiency are shown via written explanations and quantitative calculations which provide answers to a collection of problems on topics including time-independent perturbation theory, the variational method, time-dependent perturbation theory, and atomic transitions. Quantum mechanical systems considered include simple atomic systems, spin systems and harmonic oscillators.

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Multiple Choice Questions: Please write your answers in the spaces on page 5

Assume that all states are properly normalized unless otherwise specified.

1) A quantum harmonic oscillator with mass m and frequency ω is in the state

$$|\psi\rangle = \frac{1}{\sqrt{3}}(|0\rangle + |1\rangle + |2\rangle) \,.$$

If we measure the energy, what values might we obtain?
a) Any value of energy is possible.
b) Any value of energy between $\hbar\omega/2$ and $5\hbar\omega/2$ is possible, but $3\hbar\omega/2$ is most likely.
c) We will find either $\hbar\omega/2$, $3\hbar\omega/2$, or $5\hbar\omega/2$
d) The result will be $3\hbar\omega/2$.

2) For an observable $\mathcal{O}$ in a quantum system, an eigenstate of $\mathcal{O}$ is
a) A value of $\mathcal{O}$ that we might obtain in a measurement.
b) A state with a definite value for the quantity $\mathcal{O}$.
c) A state for which the value of $\mathcal{O}$ is 0.
d) A state for which the value of $\mathcal{O}$ does not change with time.

3) For a state $|\psi\rangle$ and a Hermitian operator $\hat{B}$ associated with an observable B, suppose that $\langle\psi|\hat{B}|\psi\rangle = 4$. Then we can say that
a) The observable B has a definite value of 4 in the state $|\psi\rangle$.
b) The observable B does not necessarily have a definite value of B before measuring, but we will find the value 4 if we measure B.
c) The observable B does not necessarily have a definite value of B before measuring, but the average result for a large number of measurements of B on states identical to $|\psi\rangle$ will be 4.
d) The observable B does not necessarily have a definite value of B; the quantity $\langle\psi|\hat{B}|\psi\rangle$ does not have any direct connection to the results of measurements of B.

4) If Hermitian operators $\hat{A}$ and $\hat{B}$ commute with each other, one consequence is that
a) The observables $\mathcal{A}$ and $\mathcal{B}$ are conserved.
b) The observables $\mathcal{A}$ and $\mathcal{B}$ are equal to each other for all states.
c) It is possible for a state to have a definite value of both $\mathcal{A}$ and $\mathcal{B}$, but this is not necessarily true for every state.
d) All states have definite values for $\mathcal{A}$ and $\mathcal{B}$.

5) Given a state $|\Psi\rangle$ in a quantum mechanical system with energy operator $\hat{\mathcal{H}}$, the change in the state after an infinitesimal change δt in time is

a) $\frac{\delta t}{i\hbar}\hat{\mathcal{H}}|\Psi\rangle$

b) $\langle\Psi|\hat{\mathcal{H}}|\Psi\rangle$

c) $e^{i\hat{\mathcal{H}}/\hbar\delta t}|\Psi\rangle$

d) $\hat{\mathcal{H}}\delta t$

6) For some Hermitian operator $\hat{\mathcal{O}}$ that commutes with the Hamiltonian, we can say that

a) The quantity $\hat{\mathcal{O}}|\Psi\rangle$ is proportional to the infinitesimal change in the state $|\Psi\rangle$ under time evolution.

b) All states will have definite values for $\hat{\mathcal{O}}$.

c) The ground state of the Hamiltonian will also have $\mathcal{O} = 0$.

d) A state with some expectation value for $\hat{\mathcal{O}}$ will continue to have that value.

7) For a quantum harmonic oscillator, the spacing in energy between successive energy eigenstates

a) is constant.

b) increases with increasing energy.

c) decreases with increasing energy.

d) None of the above: the allowed energies are continuous for this system.

8) If $|\Psi(\lambda)\rangle$ is an energy eigenstate for Hamiltonian $H_0 + \lambda H_1$ with energy E_0 for $\lambda = 0$, the energy of $|\Psi(\lambda)\rangle$, expressed as a power series in λ takes the form

a) $E_0 + \lambda\langle\Psi(0)|H_1|\Psi(0)\rangle + \ldots$

b) $E_0 + \lambda|\langle\Psi(0)|H_1|\Psi(0)\rangle|^2 + \ldots$

c) $E_0 + \frac{\lambda\langle\Psi(0)|H_1|\Psi(0)\rangle}{E_0 - E_1} + \ldots$

d) $E_0 + \frac{\lambda|\langle\Psi(0)|H_1|\Psi(0)\rangle|^2}{E_0 - E_1} + \ldots$

9) When the Hamiltonian is time-dependent,

a) the energy expectation value and the probabilities for finding particular energy eigenvalues in a measurement can change with time.

b) the energy expectation value is constant, but the probabilities for finding individual energy eigenvalues in a measurement can change with time.

c) the energy expectation value can change with time, but the probabilities for finding individual energy eigenvalues in a measurement are constant.

d) both the energy expectation value and the probabilities for finding individual energy eigenvalues in a measurement are unchanging in time.

10) Taking into account the spin-orbit coupling and relativistic effects in the hydrogen atom
a) completely breaks the degeneracy in the spectrum.
b) partly breaks the degeneracy, but some degeneracy remains.
c) does not break the degeneracy, but shifts the energies of states.

11) For a system with time-independent Hamiltonian H_0 in some energy eigenstate $|E_a\rangle$, if we add a time-dependent perturbation $H' = V \cos(\omega t)$, this is most likely to give rise to transitions to states
a) with higher energy E_b, such that $(E_b - E_a)/\hbar = \omega$.
b) with lower energy E_b, such that $(E_a - E_b)/\hbar = \omega$.
c) either a) or b) as long as the matrix element $\langle \psi_a | H' | \psi_b \rangle$ is nonzero.
d) none of the above: any transition must be to a state with the same energy.

12) For a certain excited state of a carbon atom, the rate for spontaneous emissions to the ground state is $A = 10^{11} s^{-1}$. At $t = 0$, we have 1,000,000 carbon atoms in this state in a vacuum with near zero temperature. Roughly how many carbon atoms would we expect to still be in this excited state after $10^{-11} s$?
a) All, or nearly all of them
b) 500,000
c) 367,000
d) 100,000
e) None, or almost none of them

13) Selection rules for electric dipole transitions are determined by
a) which energy differences in the atom match with the frequency of the background radiation.
b) which matrix elements of the electric dipole moment operator are nonzero.
c) the Pauli exclusion principle.
d) Schrödinger himself

14) For which of the following states of a two-spin system is the first spin entangled with the second?
a) $\frac{1}{\sqrt{2}}(|\uparrow\rangle \otimes |\uparrow\rangle + |\uparrow\rangle \otimes |\downarrow\rangle)$
b) $\frac{1}{\sqrt{2}}(|\downarrow\rangle \otimes |\uparrow\rangle - |\downarrow\rangle \otimes |\downarrow\rangle)$
c) $|\uparrow\rangle \otimes |\downarrow\rangle$
d) $\frac{1}{\sqrt{2}}(|\downarrow\rangle \otimes |\uparrow\rangle + |\uparrow\rangle \otimes |\downarrow\rangle)$
e) More than one of the above

15) The quantum description of the electromagnetic field is mathematically equivalent to
a) the quantum theory of a collection of harmonic oscillators.
b) the quantum mechanics of many particles interacting via a Coulomb potential.
c) the quantum mechanics of many particles interacting only by exchange forces.
d) None of the above: the electromagnetic field is an inherently classical field that interacts with quantum systems such as atoms and molecules.

16) In order to give a position-space description of the state of a quantum system with two particles in one dimension, we can use
a) two wavefunctions $\psi_1(x)$ and $\psi_2(x)$, one for each particle.
b) a single wavefunction $\psi(x_1, x_2)$ depending on two position variables.
c) two wavefunctions, $\psi_1(x_1, x_2)$ and $\psi_2(x_1, x_2)$ each depending on two variables.
d) Either a) or b).

Multiple Choice Answers:

Problem 1

Consider the state $|\Psi\rangle = |n = 3, l = 2, m = 1\rangle \otimes |s_z = \frac{1}{2}\rangle$ of an electron in a hydrogen atom.

a) For this state, if we measure J^2, what values might we obtain and what are the corresponding probabilities?

b) If we perform an infinitesimal rotation of this state around the x axis by angle θ, what is the change $\delta|\Psi\rangle$ in the state? Write your answer in the $|n\ l\ m\rangle \otimes |s_z\rangle$ basis. (*Note: don't worry about whether the rotation is clockwise or counterclockwise; just give the answer up to an overall sign*)

Problem 2

Abigail would like to estimate the ground state energy of a particle of mass m in a potential $V(x) = \lambda x^4$. She decides to use the variational method with a family of trial states $|\Psi(\omega)\rangle = |0\rangle_\omega$, i.e. the ground state of a harmonic oscillator with frequency ω (which she allows to vary). Determine the best lower bound on the energy that can be obtained using this method.

Hint: you can avoid calculating integrals by using the fact that the Hamiltonian for this system can be written as

$$H = \frac{p^2}{2m} + \lambda x^4 = (\frac{p^2}{2m} + \frac{1}{2}m\omega^2 x^2) + (\lambda x^4 - \frac{1}{2}m\omega^2 x^2),$$

where the first bracketed term is the harmonic oscillator Hamiltonian for frequency ω.

Problem 3

A particle of mass m is in the ground state of an infinite square well potential of width a. Starting at $t = 0$, the potential in the left half of the well increases at a constant rate from 0 to V in time T and then decreases back to zero at a constant rate in time T. If V is small, what is the probability that the particle will be found in the first excited state of the well at time $2T$? *Hint: the formula sheet should help.*

Problem 4

A cavity contains $3N$ molecules with two available states. There are $2N$ molecules in the ground state $|0\rangle$ with energy E_0 and N molecules in an excited state $|1\rangle$ with energy $E_1 > E_0$. The cavity contains incoherent electromagnetic radiation; the energy density per unit frequency is described by some function $\rho(\omega)$.

a) Describe the various physical processes that could cause the number of molecules in each state to change with time.

b) If the matrix elements for the components of the electric dipole operator are given by

$$\langle 1|\mathcal{P}_x|0\rangle = \langle 1|\mathcal{P}_y|0\rangle = \langle 1|\mathcal{P}_z|0\rangle = p \,,$$

what condition on $\rho(\omega)$ ensures that the number of molecules in each state will remain constant on average?

Problem 5

Consider two nearby spin half particles at fixed location. The particles sit in a magnetic field which leads to a term

$$H_1 = \frac{C}{\hbar}\left((S_1)_z + \frac{1}{2}(S_2)_z\right) \tag{1}$$

in the Hamiltonian; they also have an interaction between their magnetic moments that results in a spin-spin interaction

$$\frac{A}{\hbar^2}\vec{S}_1 \cdot \vec{S}_2 = -\frac{A}{2\hbar^2}(S_{tot}^2 - S_1^2 - S_2^2) \tag{2}$$

Here, $\vec{S}_1$ and $\vec{S}_2$ are the angular momentum operators for the two spins and $\vec{S}_{tot} = \vec{S}_1 + \vec{S}_2$

a) Assuming that A and C are positive and $A << C$ what is the energy of the ground state to the first nonzero order in A? What is the ground state in the limit that $A \to 0$?

b) Now suppose that $C << A$. In this case, what is the energy of the ground state to the first nonzero order in C? What is the ground state in the limit that $C \to 0$?

c) Make a qualitative graph of all energy levels of the system as A is varied from large negative values to large positive values for fixed C (i.e. plot E vs C for each energy eigenvalue, all on the same graph).