Exam D answers.

1 a) spin-orbit coupling $\longrightarrow \vec{L}\cdot\vec{S}\cdot c_1$

relativistic corrections to energy $\longrightarrow p^4 \cdot c_2$

b) $\langle 311|\hat{x}|100\rangle$

$\langle 311|\hat{y}|100\rangle$

$\langle 311|\hat{z}|100\rangle$

Need energy density $\rho(\omega_0)$ at $\omega_0 = \dfrac{E_3 - E_1}{\hbar}$

c) Transitions due to matrix elements in H'_{EM} beyond the dipole approximation.

d) - Normalize to find $A(b)$

- Calculate $\langle\psi|H|\psi\rangle = \int_{-\infty}^{\infty} dx\, \psi_b(x)\left[-\frac{\hbar^2}{2m}\frac{d}{dx^2} + V(x)\right]\psi_b(x)$

- Minimize over b.

- This is an upper bound.

2. a) Have:

$$H = -\vec{\mu}\cdot\vec{B}$$

$$= -\frac{q}{2m} B S_z \qquad (\text{since } g=1)$$

$\therefore$ Energies (spin 1) are $\underset{E_{m=-1}}{\frac{qB\hbar}{2m}}$, $\underset{E_{m=0}}{0}$, $\underset{E_{m=1}}{-\frac{qB\hbar}{2m}}$

b) $H' = \alpha\left(S_x^2 - S_y^2\right) = \frac{\alpha}{2}\left(S_+^2 + S_-^2\right)$

$\delta E_m = \langle m|H'|m\rangle = 0$ for each state. to 1st order.

Need 2nd order P.T.

$$\delta^{(2)} E_{m=1} = \sum_{m'} \frac{|\langle m' | H' | m \rangle|^2}{E_{m=1} - E_{m'}} = \frac{|\langle m'=-1 | \frac{\alpha}{2} S_-^2 | m=1 \rangle|^2}{E_{m=1} - E_{m'=-1}}$$

$$S_- | m \rangle = \hbar \sqrt{s(s+1) - m(m-1)}$$

$$\delta^{(2)} E_{m=1} = \frac{\alpha^2 \hbar^4}{qB\hbar/m} = \frac{m\alpha^2\hbar^3}{qB}$$

c) Full Hamiltonian is

$$H' = \begin{pmatrix} -\frac{qB\hbar}{2m} & 0 & \alpha\hbar^2 \\ 0 & 0 & 0 \\ \alpha\hbar^2 & 0 & \frac{qB\hbar}{2m} \end{pmatrix} \quad \text{in } |s\ m_z\rangle \text{ basis.}$$

Ground state energy: $-\sqrt{\alpha^2\hbar^4 + \frac{q^2B^2\hbar^2}{4m^2}}$

3. Have:

$$H' = \begin{pmatrix} 0 & 0 & \alpha\hbar^2 \\ 0 & 0 & 0 \\ \alpha\hbar^2 & 0 & 0 \end{pmatrix}$$

Eigenstates: $\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$: $E = 0$ $\quad \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$: $E = \alpha\hbar^2$ $\quad \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$: $E = -\alpha\hbar^2$

b) Ground state: $\frac{1}{\sqrt{2}} | m = 1 \rangle - \frac{1}{\sqrt{2}} | m = -1 \rangle$

$\therefore S_z = \frac{\hbar}{2}$ w. prob $\frac{1}{2}$ $\quad -\frac{\hbar}{2}$ w. prob $\frac{1}{2}$

4: Have: $P_{1\to n} = \left| \frac{1}{\hbar} \int_0^{\frac{\pi}{\Omega}} dt\, e^{i\Omega(n-1)t} \langle n | A x^3 | 1 \rangle \right|^2$

$$x^3 = \left(\sqrt{\frac{\hbar}{2m\omega}}\right)^3 (a + a^\dagger)^3$$

Have: $\langle 4 | x^3 | 1 \rangle = 2\sqrt{6}$

$\langle 2 | x^3 | 1 \rangle = 6\sqrt{2}$

$\langle 0 | x^3 | 1 \rangle = 3$

$P_{1\to 4} = \frac{24 A^2}{\hbar^2 \Omega^2} \cdot \frac{4}{9} = \frac{32}{3} \frac{A^2}{\hbar^2\Omega^2}$ (prob of getting $\hbar\Omega \cdot \frac{9}{2}$)

$P_{1\to 2} = \frac{288 A^2}{\hbar^2 \Omega^2}$ (prob of getting $\hbar\Omega \cdot \frac{5}{2}$)

$P_{1\to 0} = \frac{36 A^2}{\hbar^2 \Omega^2}$ (prob of getting $\hbar\Omega \cdot \frac{1}{2}$)

b) require: $P \ll 1 \therefore A \ll \hbar\Omega$.

6. a) Lowest: $\frac{1}{\sqrt{2}}\left(|000\uparrow\rangle \otimes |000\downarrow\rangle - |000\downarrow\rangle \otimes |000\uparrow\rangle\right)$ $E = \frac{3\hbar\omega}{2}$

$n_x\ n_y\ n_z\ s_z$

Next: can have $\left[|100\rangle, |001\rangle \text{ or } |010\rangle \otimes |\uparrow\rangle \text{ or } |\downarrow\rangle \right] \otimes \left[|000\uparrow\rangle \text{ or } |000\downarrow\rangle \right]$

antisymmetrize.

$E = \frac{5\hbar\omega}{2}$, 12 possible states in basis.

b) Have zero orbital angular momentum (spherically symmetric wavefunction for $n_x = n_y = n_z = 0$), spin state $\frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) = |J=0\ M=0\rangle$

$\therefore \langle J^2 \rangle = 0$

c) Have: $\frac{1}{\sqrt{2}}\left(|000\downarrow\rangle\otimes|100\downarrow\rangle - |100\downarrow\rangle\otimes|000\downarrow\rangle\right)$

$\frac{1}{\sqrt{2}}\left(|000\downarrow\rangle\otimes|010\downarrow\rangle - |010\downarrow\rangle\otimes|000\downarrow\rangle\right)$

$\frac{1}{\sqrt{2}}\left(|000\downarrow\rangle\otimes|001\downarrow\rangle - |001\downarrow\rangle\otimes|000\downarrow\rangle\right)$

ground states $E = \frac{5\hbar\omega}{2}$

Next level: antisymmetrization of

$|000\downarrow\rangle\otimes|200\downarrow\rangle$
$|000\downarrow\rangle\otimes|020\downarrow\rangle$
$|000\downarrow\rangle\otimes|002\downarrow\rangle$
$|100\downarrow\rangle\otimes|010\downarrow\rangle$
$|100\downarrow\rangle\otimes|001\downarrow\rangle$
$|010\downarrow\rangle\otimes|001\downarrow\rangle$

6 states $E = \frac{7\hbar\omega}{2}$

d) Spin part: $S_{tot} = 1$ $(S_z)_{tot} = -1$

Orbital part: need L_z eigenvalues for $|100\rangle$ states
$|010\rangle$
$|001\rangle$

Find: $|001\rangle$: $L_z = 0$

$\frac{1}{\sqrt{2}}\left(|100\rangle + i|010\rangle\right)$ $L_z = \hbar$

$\frac{1}{\sqrt{2}}\left(|100\rangle - i|010\rangle\right)$ $L_z = -\hbar$

$\therefore$ $\ell = 1$ states

Call these $|\ell=1\ m\rangle$. Full states are: $|\ell=1\ m\rangle\otimes|s=1\ s_z=-1\rangle$

Use CG table to write each in $J\ M$ basis & read off probabilities for J^2 to calculate $\langle J^2\rangle$

7: Have $\frac{d}{dt}\langle\psi|\hat{O}|\psi\rangle = \left(\frac{d}{dt}\langle\psi|\right)\hat{O}|\psi\rangle + \langle\psi|\hat{O}\left(\frac{d}{dt}|\psi\rangle\right)$

$= \frac{i}{\hbar}\langle\psi|\hat{H}\hat{O}|\psi\rangle - \frac{i}{\hbar}\langle\psi|\hat{O}\hat{H}|\psi\rangle$

$= \frac{i}{\hbar}\langle\psi|[\hat{H},\hat{O}]|\psi\rangle = 0.$