Exam C answers:

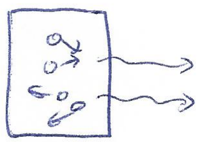
a) Spin orbit coupling (interaction of moving magnetic dipole moment of electron) w. electric field of proton.

+ Relativistic corrections (using complete formula for E in terms of p)

b) Have $\left|\tfrac{3}{2}\ \tfrac{1}{2}\right\rangle = \sqrt{\tfrac{3}{5}}\left|m=1\ s_z=-\tfrac{1}{2}\right\rangle - \sqrt{\tfrac{2}{5}}\left|m=0\ s_z=\tfrac{1}{2}\right\rangle$

$$\langle L_z\rangle = \hbar\left(\tfrac{3}{5}\right)\cdot 1 + \left(\tfrac{2}{5}\right)\hbar\cdot 0$$
$$= \tfrac{3}{5}\hbar$$

c) - Doppler shift



emitted light from atoms redshifted/blue shifted from their motion relative to detector.

- Natural line width: eigenstates are actually combinations of true eigenstates of full system of atom + EM field. $\therefore$ intial state has range of possible energies.

2 a Label states as $|n\ s_z\rangle$ for a single particle. Then:

lowest: $\boxed{\tfrac{1}{\sqrt{2}}\left(|n=1,\uparrow\rangle\otimes|n=1,\downarrow\rangle - |n=1,\downarrow\rangle\otimes|n=1,\uparrow\rangle\right)}$ deg. 1 energy: $\dfrac{\pi^2\hbar^2}{ma^2}$

next:
$\tfrac{1}{\sqrt{2}}\left(|n=1,\uparrow\rangle\otimes|n=2,\uparrow\rangle - |n=2,\uparrow\rangle\otimes|n=1,\uparrow\rangle\right)$
$\tfrac{1}{\sqrt{2}}\left(|n=1,\uparrow\rangle\otimes|n=2,\downarrow\rangle - |n=2,\downarrow\rangle\otimes|n=1,\uparrow\rangle\right)$
$\tfrac{1}{\sqrt{2}}\left(|n=1,\downarrow\rangle\otimes|n=2,\uparrow\rangle - |n=2,\uparrow\rangle\otimes|n=1,\downarrow\rangle\right)$
$\tfrac{1}{\sqrt{2}}\left(|n=1,\downarrow\rangle\otimes|n=2,\downarrow\rangle - |n=2,\downarrow\rangle\otimes|n=1,\downarrow\rangle\right)$

deg. 4. energy: $\dfrac{5\pi^2\hbar^2}{2ma^2}$

c): ground states

b) lowest: $\boxed{\tfrac{1}{\sqrt{2}}\left(|n=1,\downarrow\rangle\otimes|n=2,\downarrow\rangle - |n=2,\downarrow\rangle\otimes|n=1,\downarrow\rangle\right)}$ energy $\dfrac{5\pi\hbar^2}{2ma^2}$ deg: 1

next: $\tfrac{1}{\sqrt{2}}\left(|n=1,\downarrow\rangle\otimes|n=3,\downarrow\rangle - |n=3,\downarrow\rangle\otimes|n=1,\downarrow\rangle\right)$ energy $\dfrac{5\pi\hbar^2}{ma^2}$ deg 1

3 a) For any state $|\psi\rangle$, normalized.

$$E_0 \leqslant \langle\psi|H|\psi\rangle$$

ground state energy

b) Normalize: $A^2+B^2=1$

$$E_0 \leqslant \alpha\langle\psi| S_x+S_y|\psi\rangle$$

$$= \alpha\left(\frac{1-i}{2}\right)\langle\psi|S_+|\psi\rangle + \alpha\left(\frac{1+i}{2}\right)\langle\psi|S_-|\psi\rangle$$

OR use $S_x = \frac{\hbar}{2}\begin{pmatrix}0&1\\1&0\end{pmatrix}$ $S_y = \frac{\hbar}{2}\begin{pmatrix}0&i\\i&0\end{pmatrix}$ in S_z basis.

get $E_0 \leqslant \alpha(A^*B)\cdot\hbar$

Minimize over AB with $A^2+B^2=1$. Get $A=\frac{1}{\sqrt{2}}$ $B=-\frac{1}{\sqrt{2}}$ or $A=\frac{-1}{\sqrt{2}}$ $B=\frac{1}{\sqrt{2}}$

$$\therefore E_0 \leqslant -\frac{\alpha\hbar}{2}$$

c) Exact: $E_0 = -\frac{\sqrt{2}}{2}\hbar\alpha$ (write $H = \sqrt{2}\alpha S_{\hat{n}}$) eigenvalues $\pm\frac{\hbar}{2}$.

$$\hat{n} = \frac{\hat{x}+\hat{y}}{\sqrt{2}}$$

4. See homework 11 solutions.

5. See homework 12 solutions: $\text{Rate} = \dfrac{\omega^2 e^2}{6\pi\varepsilon_0 m c^3}$

6. See hw. 7 solutions Have $\delta E_1 = \dfrac{k\hbar}{2m\omega}$

Reliable if $k \ll 2m\omega^2$

Exact: $\frac{1}{2}\hbar\omega + \frac{1}{2}\hbar\omega\sqrt{1+\frac{2k}{m\omega^2}}$