ANSWERS

Exam A:

a) We can have: $\ell = 1$, $m = \pm 1, 0$, $s_z = \pm \frac{1}{2}$ OR $\ell = 0$, $m = 0$, $s_z = \pm \frac{1}{2}$

In the (ℓ, J, M) basis, we can have:

$\ell = 1$, $J = \frac{3}{2}$, $M = -\frac{3}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{3}{2}$

$\ell = 1$, $J = \frac{1}{2}$ $M = -\frac{1}{2}, \frac{1}{2}$

$\ell = 0$, $J = \frac{1}{2}$ $M = \pm\frac{1}{2}$

b) It depends on ~~the energy difference $E_b - E_a$~~, the energy density of radiation at frequency $\omega = \frac{|E_b - E_a|}{\hbar}$, and the matrix element $\langle \psi_b | \wp_i | \psi_a \rangle$ of the electric dipole moment operator between the two states

c) Have $n=6$. Stays at $n=6$ by adiabatic theorem but $a \to 3a$ so

$$E \to \frac{1}{9} E = \frac{2\pi^2 \hbar^2}{m a^2}$$

2. Need 2nd order pert. theory.

Have $\delta E^{(2)} = -\frac{11}{8} \lambda^2 \frac{\hbar^2}{m^3 \omega^4}$

Reliable if $|\delta E^{(2)}| \ll |E_0|$

$$\frac{\lambda^2 \hbar^2}{m^3 \omega^4} \ll \hbar\omega$$

4.

$$\psi_1(x) = \langle x | 1 \rangle = \langle x | a^\dagger | 0 \rangle = \sqrt{\frac{m\omega}{2\hbar}} \langle x | x | 0 \rangle - \frac{i}{\sqrt{2m\hbar\omega}} \langle x | p | 0 \rangle$$

$$= \sqrt{\frac{m\omega}{2\hbar}} \times \psi_0(x) - \frac{\hbar}{\sqrt{2m\hbar\omega}} \frac{d}{dx} \psi_0(x)$$

use $\langle x | p | \psi \rangle = \frac{\hbar}{i} \frac{d}{dx} \langle x | \psi \rangle$

Answer: $\frac{1}{\pi^{1/4}} \left(\frac{m\omega}{\hbar} \right)^{3/4} x \sqrt{2}\, e^{-\frac{m\omega x^2}{2\hbar}}$.

5. Hamiltonian is

$$H' = -\vec{\mu}\cdot\vec{B}$$
$$\approx -\frac{e}{m}\vec{S}\cdot\vec{B}$$
$$= -\frac{e}{m}(B)(at\cdot S_x + bt^2 S_y + ct^3 S_z)$$

Use time-dep. pert. theory:

$$P \approx \left|\frac{1}{\hbar}\int_0^T H'_{ba}(t')\,dt'\right| \qquad \text{have: } (\omega_{ba}=0)$$

$$H'_{ba} = \langle\downarrow|H'|\uparrow\rangle$$
$$= -\frac{e}{m}B\left(at\cdot\frac{\hbar}{2} + bt^2\cdot\frac{\hbar}{2}i\right)$$
$$\therefore P \approx \left|\frac{1}{\hbar}\frac{eB}{m}\left(\hbar\frac{aT^2}{4} + i\,\frac{bT^3}{6}\right)\right|^2$$
$$= \left(\frac{eB}{m}\right)^2\left[\left(\frac{aT^2}{4}\right)^2 + \left(\frac{bT^3}{6}\right)\right]^2$$

6. a) Eigenstates: $|l\ m\rangle = |1\ 1\rangle$ energy $\alpha\hbar^2$

$|1\ 0\rangle$ energy 0

$|1\ -1\rangle$ energy $\alpha\hbar^2$

b) energy shift for $|1\ 0\rangle$ is $\delta E = \langle 1\ 0|\beta L_x^2|1\ 0\rangle$ ← calculate using $L_x = \frac{1}{2}(L_+ + L_-)$

$= \beta\hbar^2$

shifts for $|1\ 1\rangle, |1\ -1\rangle$ are eigenvalues of

$$\begin{pmatrix} \langle 1\,1|\beta L_x^2|1\,1\rangle & \langle 1\,1|\beta L_x^2|1\,-1\rangle \\ \langle 1\,-1|\beta L_x^2|1\,1\rangle & \langle 1\,-1|\beta L_x^2|1\,-1\rangle \end{pmatrix} \rightarrow \delta E = 0, \beta\hbar^2$$

c) For $H = \alpha(L_x^2 + L_z^2) = \alpha(\vec{L}^2 - L_y^2)$ eigenvalues are same as for $L^2 - L_z^2$

i.e. $(\hbar^2 l(l+1) - \hbar^2 m^2)\cdot\alpha \quad l=1 \quad m=\pm 1, 0$