

## Physics 402 Quiz 1, February 1, 2018

1) For a harmonic oscillator system, which of the following is zero?

- a)  $\langle 0|x^2|0\rangle$
- b)  $\langle 2|x^2|2\rangle$
- c)  $\langle 2|a^2 + (a^\dagger)^2|2\rangle$
- d)  $\langle 0|a^2(a^\dagger)^2|0\rangle$
- e) 17

2) We add a perturbation  $\lambda x^2$  to a harmonic oscillator Hamiltonian. The first order shift in the ground state  $|0\rangle$  is a linear combination  $c_n|n\rangle$  of the original harmonic oscillator eigenstates. For how many  $n$  is  $c_n$  not zero?

- a) 1
- b) 2
- c) 3
- d) 4
- e)  $\infty$

3) The first order shift in the energy of the ground state for the perturbation in the previous question is

a) 
$$\lambda\langle 0|x^2|0\rangle$$

b) 
$$\lambda\sum_n \frac{\langle 0|x^2|n\rangle}{E_n - E_0}$$

c) 
$$\lambda\sum_n \frac{\langle n|x^2|0\rangle}{E_n - E_0}$$

d) 
$$\lambda\langle 1|x^2|1\rangle$$

e) zero

- 4) A Hamiltonian  $H_0$  has two states  $|A\rangle$  and  $|B\rangle$  with the same energy  $E$ . Which of the following is true about the system if we add a small perturbation  $\lambda H_1$  to the Hamiltonian?
- The perturbed system will have energy eigenstates with energies close to  $E$  and these will be close to  $|A\rangle$  and  $|B\rangle$ .
  - The perturbed system will have energy eigenstates with energies close to  $E$  but these won't necessarily be close to  $|A\rangle$  and  $|B\rangle$ .
  - The perturbed system will have energy eigenstates close to  $|A\rangle$  and  $|B\rangle$  but their energies won't necessarily be close to  $E$ .
  - The perturbed system will usual have neither energy eigenstates with energies close to  $E$  nor energy eigenvectors close to  $|A\rangle$  and  $|B\rangle$ .

5) A particle is in the ground state of an infinite square well potential located on the interval  $[0, L]$ ; call the wavefunction for this particle  $\psi_0(x)$ . If we perturb the system by adding a potential  $\lambda V(x)$ , the shift in ground state energy will be equal to

- $V(L/2)$
- $\int_0^L dx V(x)$
- $\int_0^L dx V(x) \psi_0(x)$
- $\int_0^L dx V(x) |\psi_0(x)|^2$
- All of the above are correct

6) If we use the formula

$$E_n^2 = \sum_{m \neq n} \frac{|\langle m | H_1 | n \rangle|^2}{E_n - E_m} \quad (1)$$

to calculate the second order shift in energy for the ground state (i.e. the lowest energy  $n = 0$  state) of a quantum system, the result will always be

- positive
- negative
- zero

*Hint: the answer is apparent from the formula.*

**Please fill in:**

Answers

1	2	3	4	5	6
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