Physics 402 Quiz 1, Feb 5, 2020

Name/Student Number:

1) Which of the states below is physically equivalent to the (unnormalized) state $i|\uparrow\rangle - 2|\downarrow\rangle$?

a) $i|\uparrow\rangle + 2|\downarrow\rangle$

b) $\frac{1}{3}|\uparrow\rangle - \frac{2}{3}i|\downarrow\rangle$

c) $\frac{1}{\sqrt{5}}|\uparrow\rangle + \frac{2i}{\sqrt{5}}|\downarrow\rangle$ $\times \frac{i}{\sqrt{5}}$ so equivalent

d) $-i|\uparrow\rangle - 2|\downarrow\rangle$

e) none of the above

2) For a quantum system with a two-dimensional Hilbert space, observables Z and X are represented by $Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ and $X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ in the basis of Z eigenstates. For this system, we can say that

$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ or $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$

a) There are no states with a definite value of either Z or X. neither have definite X value

b) Certain states have a definite value for Z or X, but no states have a definite value for both Z and X.

c) Most states do not have a definite value for both Z and X but certain special states do.

d) All states have a definite value for both Z and X.

3) At time $t = 0$, the state of a quantum system with a time-independent Hamiltonian is a linear combination of energy eigenstates with different energies:

Energy conserved!

$$|\Psi(t=0)\rangle = \sqrt{\frac{1}{3}}|E_1\rangle + \sqrt{\frac{2}{3}}|E_2\rangle \ . \qquad (1)$$

$\langle E \rangle$ and $p(E_i)$ constant in time

For this state,

a) the probability of finding E_1 in a measurement of energy doesn't change with time, and the expectation value of energy doesn't change with time.

b) the probability of finding E_1 in a measurement of energy doesn't change with time, but the expectation value of energy can change with time.

c) the probability of finding E_1 in a measurement of energy can change with time, but the expectation value of energy doesn't change with time.

d) both the probability of finding E_1 in a measurement of energy and the expectation value of energy can change with time.

4) For a quantum system with operator $\hat{\mathcal{O}}$ associated with some observable $\mathcal{O}$, we can say that $\hat{\mathcal{O}}|\Psi\rangle$ is

a) the state $|\Psi\rangle$ after a measurement of $\mathcal{O}$. ✗ → this will be $|\lambda_i\rangle$ (some eigenstate of $\mathcal{O}$)

b) proportional to the change in the state $|\Psi\rangle$ under the infinitesimal physical transformation associated with $\hat{\mathcal{O}}$.

$|\Psi\rangle \to (1 + i\varepsilon\hat{\mathcal{O}})|\Psi\rangle$ so change is $i\varepsilon\hat{\mathcal{O}}|\Psi\rangle$

c) always just equal to the state $|\Psi\rangle$ again but multiplied by a phase. → true for eigenstate of $\mathcal{O}$, but not generally

d) the expectation value of $\mathcal{O}$ in the state $|\Psi\rangle$. → this is $\langle\Psi|\mathcal{O}|\Psi\rangle$

5) If $[\hat{\mathcal{O}}, H] = 0$ for an operator $\hat{\mathcal{O}}$ in a system with time-independent Hamiltonian H, which of the following is **not** necessarily true?

a) Every energy eigenstate has a definite value for the observable $\mathcal{O}$ associated with $\hat{\mathcal{O}}$. Not necessarily:

b) There is a basis of states for which both $\mathcal{O}$ and H are represented by diagonal matrices. ✓

c) For any state, the probabilities for various measurement outcomes of $\mathcal{O}$ do not change with time. ✓ $\mathcal{O}$ is conserved

d) The operator $e^{ia\hat{\mathcal{O}}}$ is a unitary matrix representing a physical transformation that is a symmetry of the system. ✓ transform associated w $\mathcal{O}$ is a symmetry

6) Physical transformations such as rotations are represented by unitary operators acting on the Hilbert space of a quantum system. Which of the following is **not** a property of unitary operators?

a) They are linear maps from the Hilbert space to itself.

b) They preserve the inner product between states: if $|A'\rangle = U|A\rangle$ and $|B'\rangle = U|B\rangle$, then $\langle A'|B'\rangle = \langle A|B\rangle$.

c) They have an orthogonal basis of eigenvectors with real eigenvalues. → true for Hermitian ops, but not unitaries (eigenvalues are complex phases)

d) They map normalized states to normalized states.

Answers

1	2	3	4	5	6
C	B	A	B	A	C