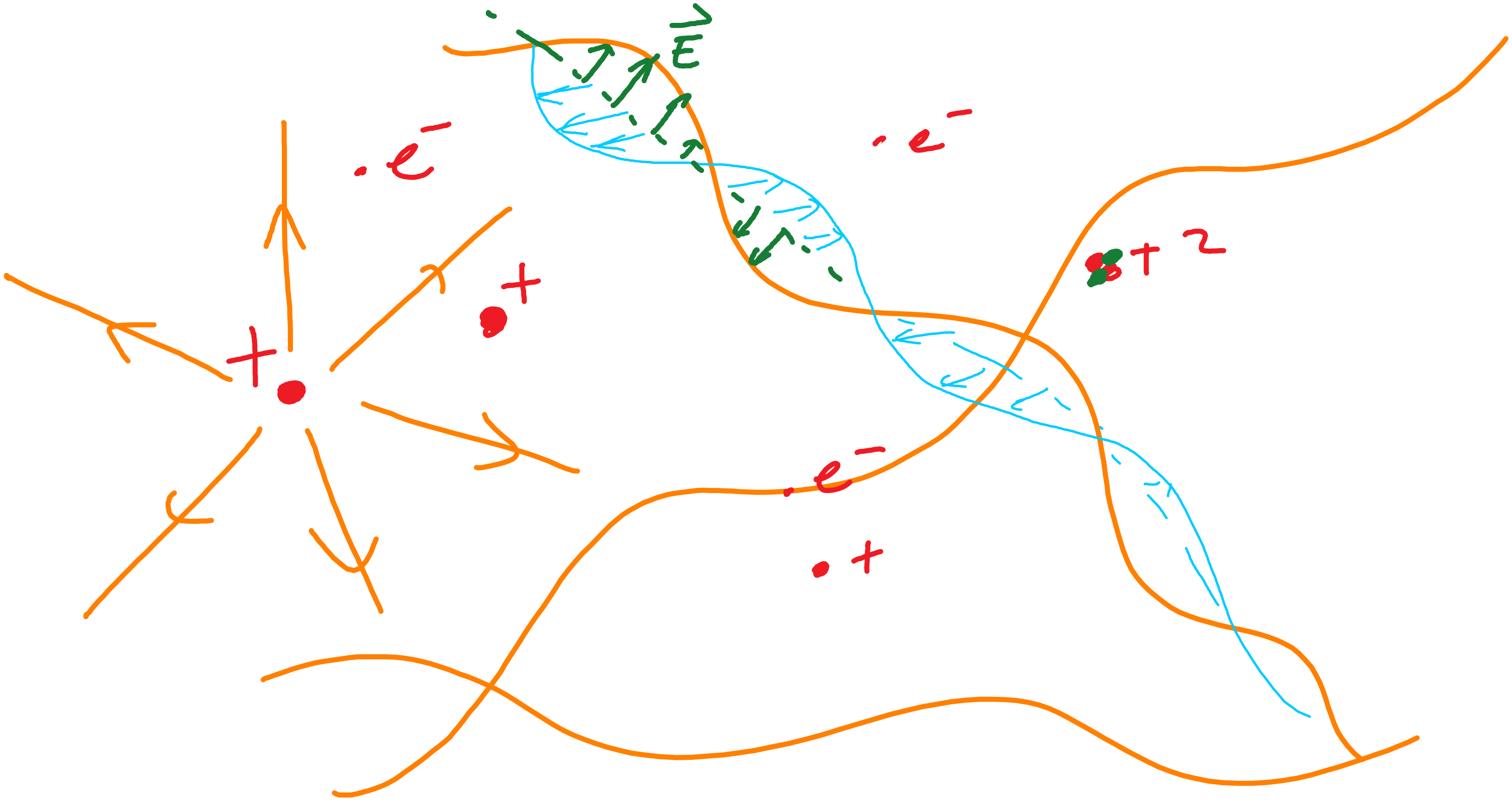
Quantum Field Theory

Microscopic picture of matter: particles + fields



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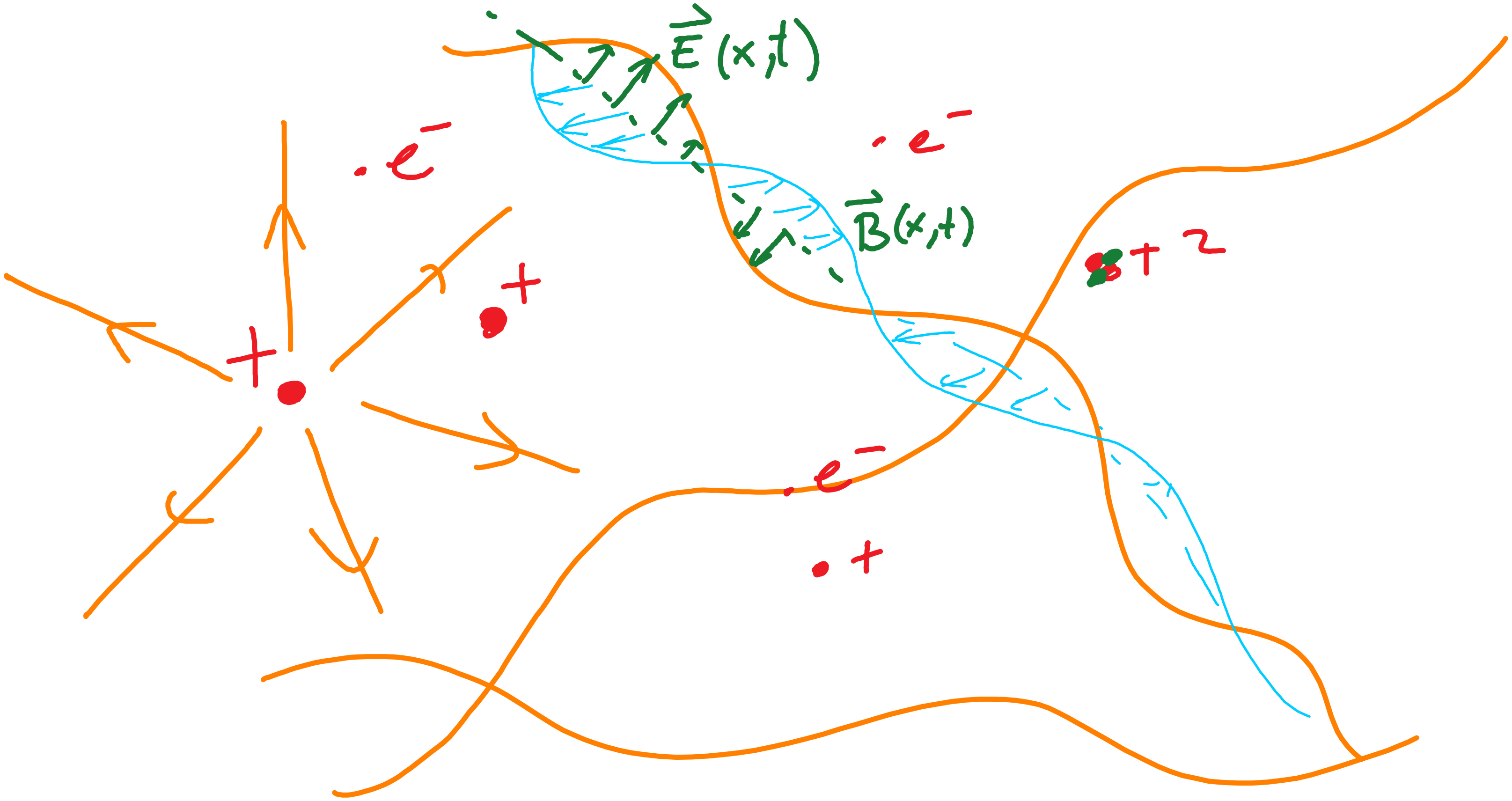
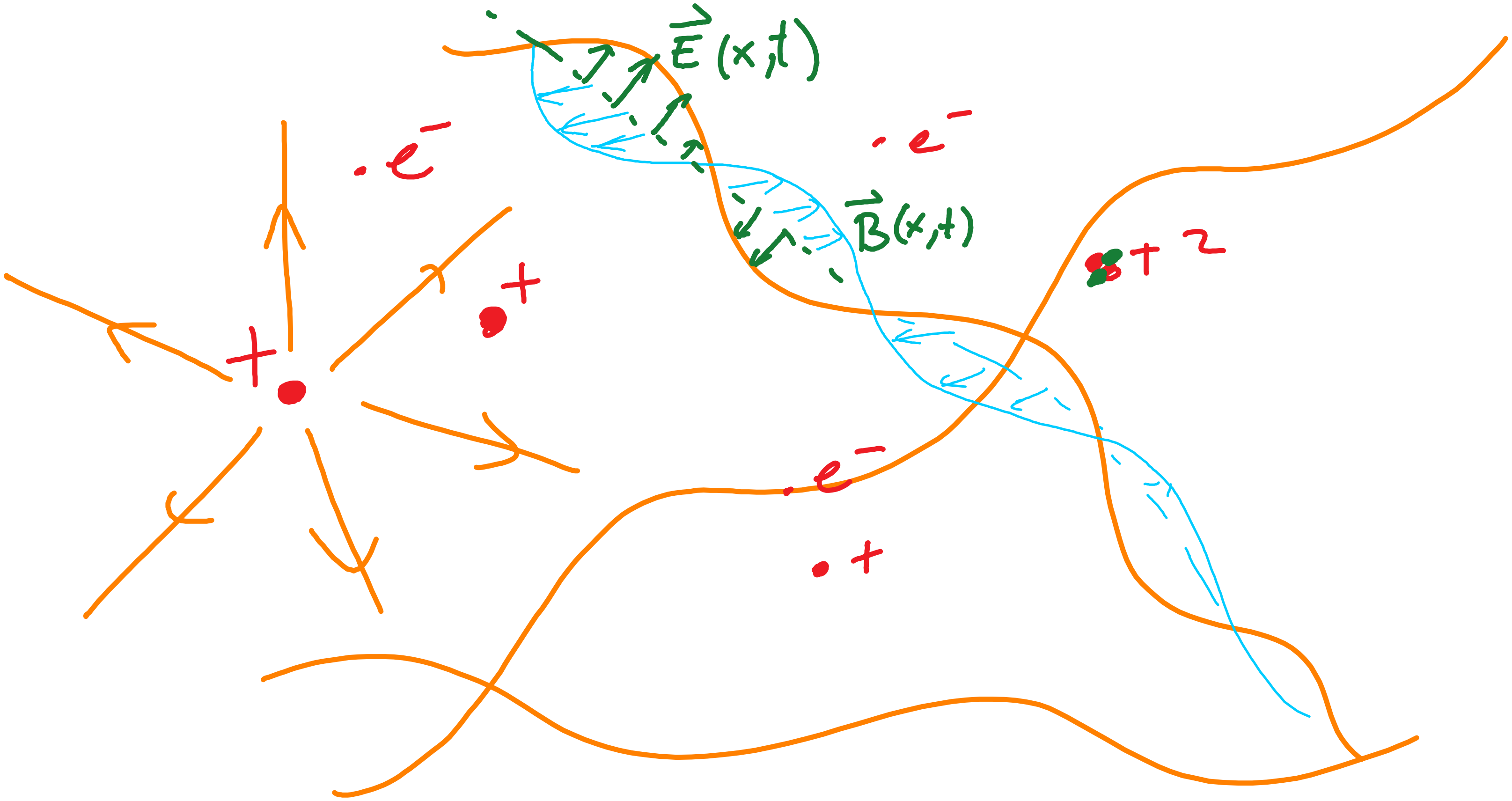


Principles of QM also apply to fields:

- classical configurations $\vec{E}(x)$ become basis elements $|\vec{E}(x)\rangle$

g eneral states are superpositions

Microscopic picture of matter: particles + fields



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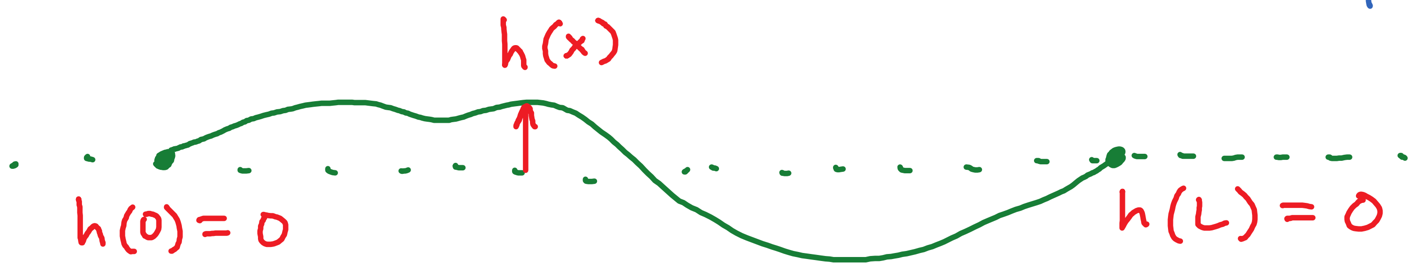
- classical configurations $\vec{E}(x)$ become basis elements $|\vec{E}(x)\rangle$
- general states are superpositions

What are the physical consequences?

FIELD THEORY = physical system whose classical configurations are FIELDS i.e. functions of space & time

Simplest example: a 1D field (e.g. guitar string, plane EM waves)



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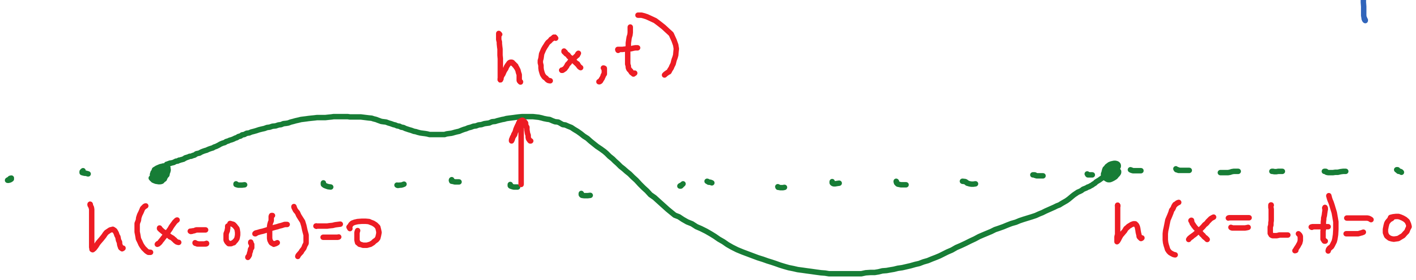
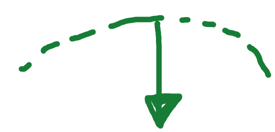


Obeys wave equation: $\dfrac{\partial^2 h}{\partial t^2} = v^2 \dfrac{\partial^2 h}{\partial x^2}$



Energy: $E = \displaystyle\int_0^L dx \left\{ \frac{1}{2}\rho \left(\frac{\partial h}{\partial t}\right)^2 + \frac{1}{2}\rho v^2 \left(\frac{\partial h}{\partial x}\right)^2 \right\}$

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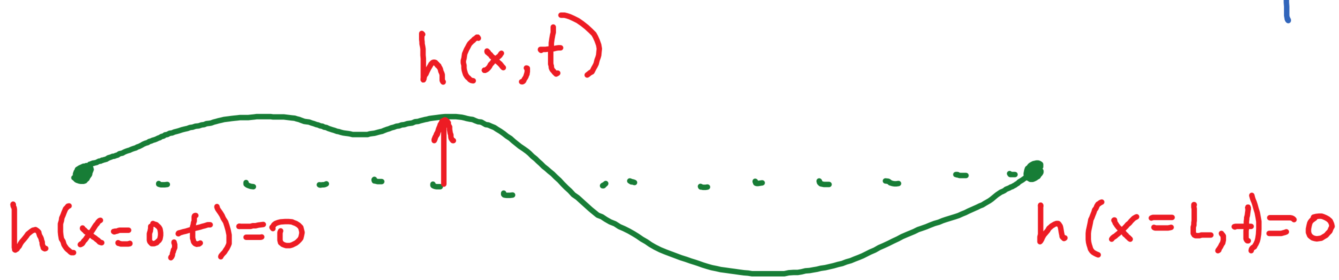

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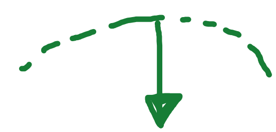

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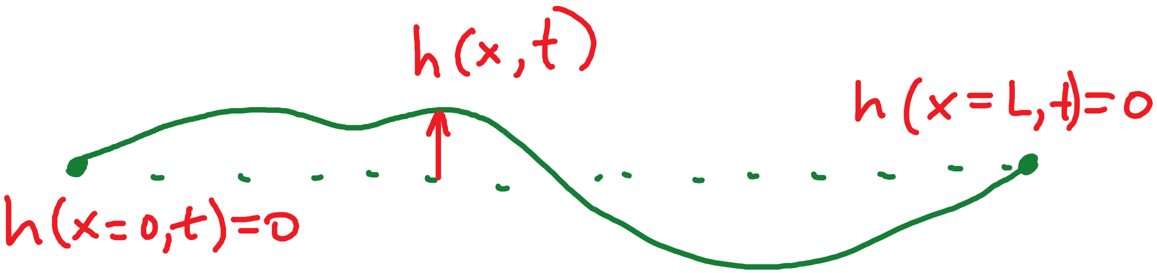
What is the quantum physics?

Trick: write $h(x,t) = \sum_k h_k(t) \sin\left(\frac{k\pi x}{L}\right)$ Fourier series



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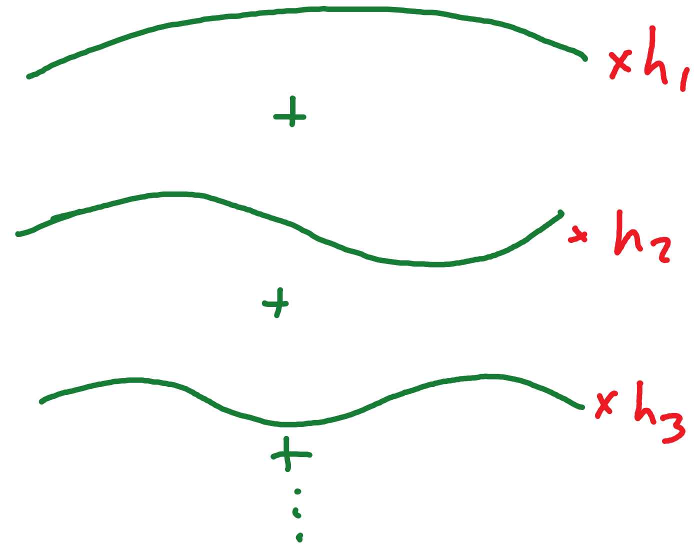

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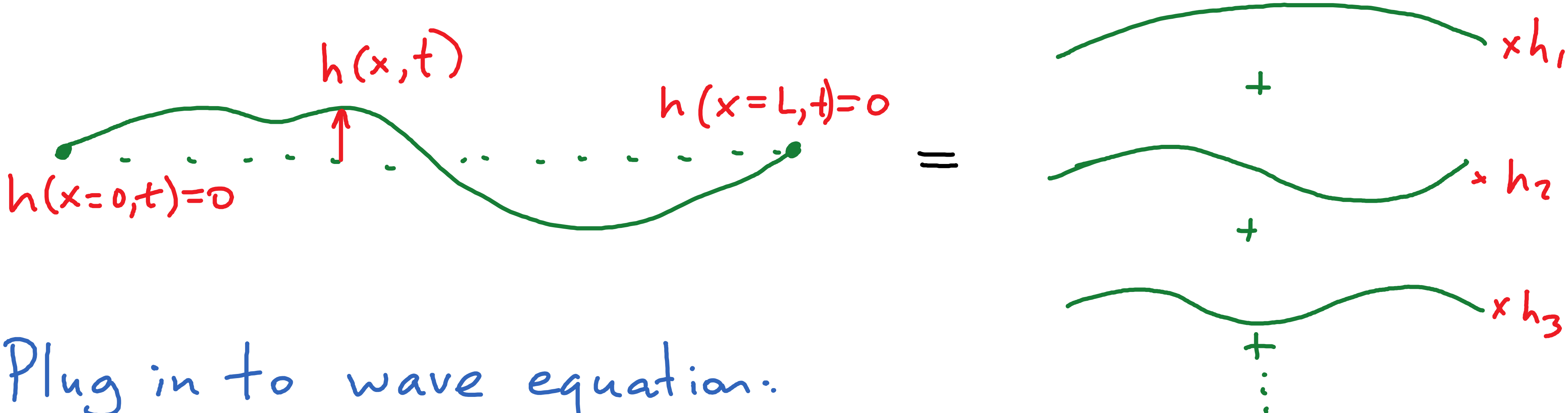

Plug in to wave equation:

$$\frac{\partial^2 h}{\partial t^2} = v^2 \frac{\partial h}{\partial x^2} \longrightarrow \sum_k \left(\frac{d^2 h_k}{dt^2} + v^2 \left(\frac{k\pi}{L}\right)^2 h_k \right) \sin\left(\frac{k\pi x}{L}\right) = 0$$

$$\Rightarrow \quad \frac{d^2 h_k}{dt^2} = -v^2 \left(\frac{k\pi}{L}\right)^2 h_k$$

* h_k like harmonic oscillator with $\omega_k = v\left(\frac{k\pi}{L}\right)$ *

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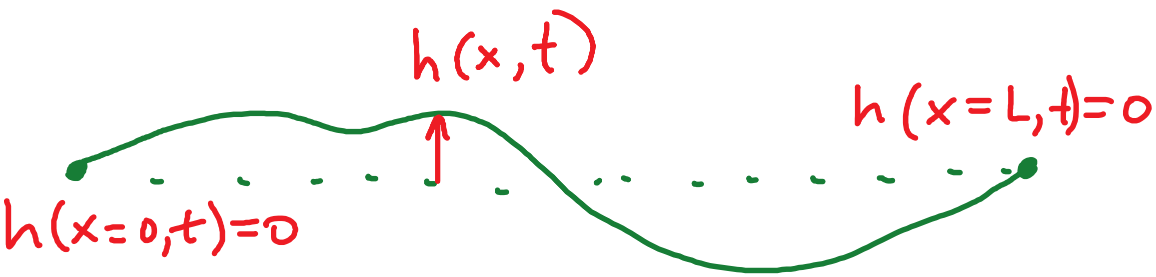

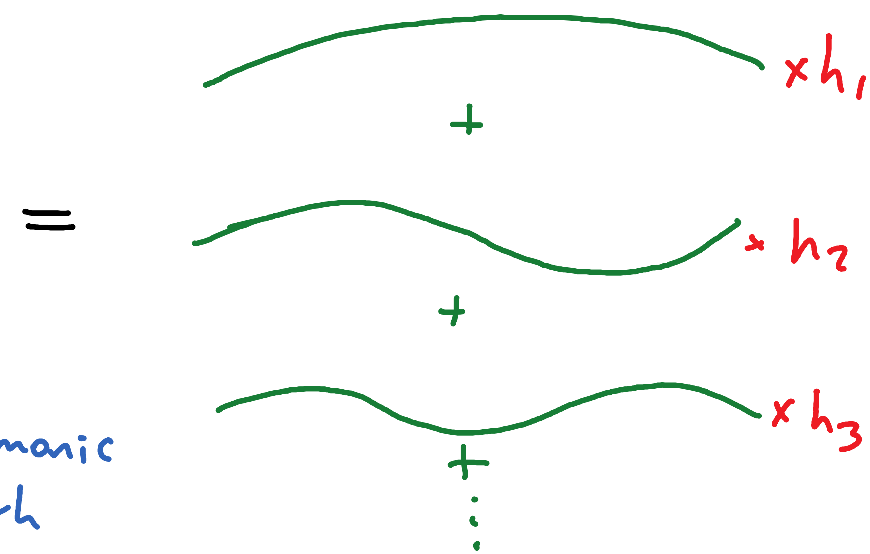

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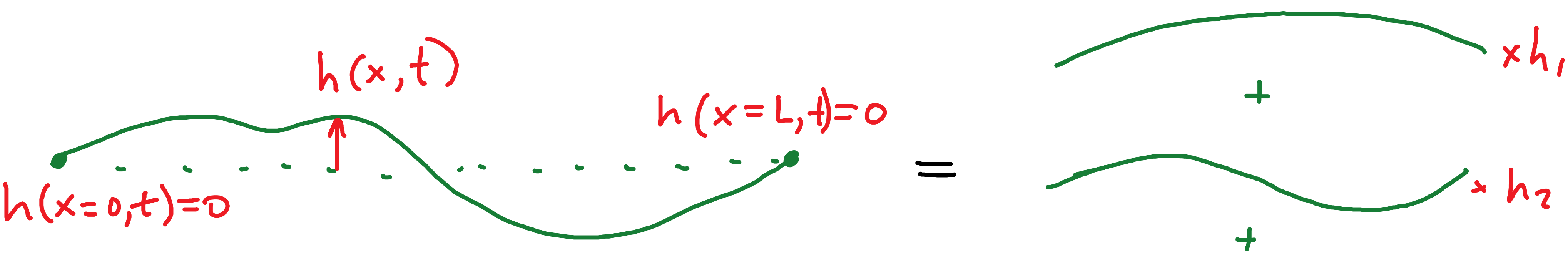

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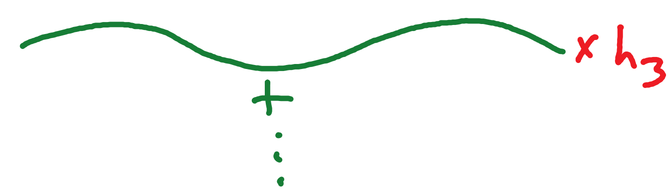


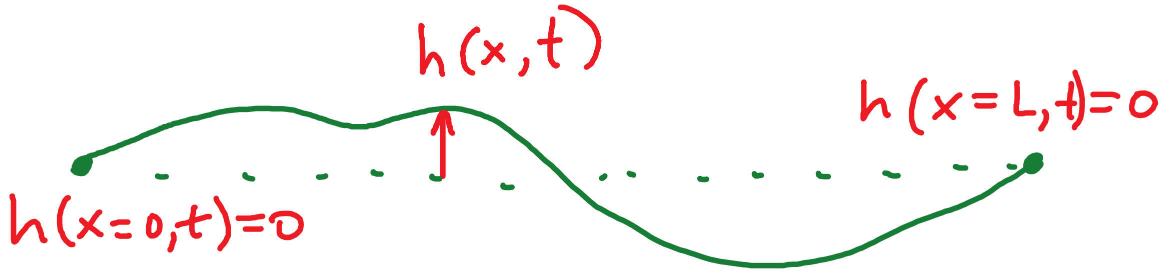
* h_k like harmonic oscillator with $\omega_k = v\left(\frac{k\pi}{L}\right)$ *

Energy is:

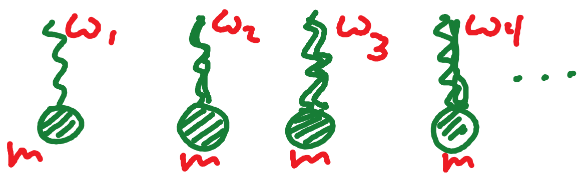
$$E = \sum_n \left(\frac{L}{4}\rho \dot{h}_k^2 + \frac{L}{4}\rho \cdot v \cdot \left(\frac{k\pi}{L}\right)^2 h_k^2\right)$$

$$= \sum_n \left(\frac{1}{2} m \dot{h}_k^2 + \frac{1}{2} m \omega_k^2 h_k^2\right) \qquad m \equiv \frac{L}{2}\rho$$

* Same as for collection of harmonic oscillators *

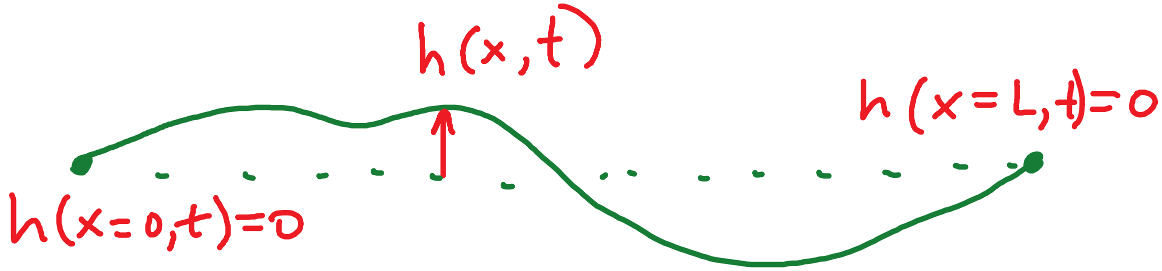


exactly equivalent to



$$m = \frac{\rho L}{2}$$

$$\omega_k = v\left(\frac{k\pi}{L}\right)$$



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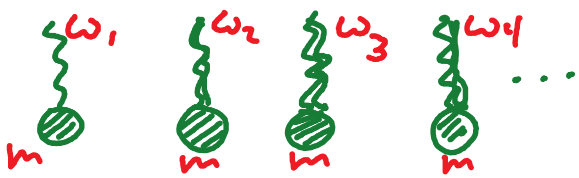


$$m = \frac{\rho L}{2}$$

$$\omega_k = v\left(\frac{k\pi}{L}\right)$$

Quantum: each harmonic oscillator has states $|n\rangle$ with energies $\hbar\omega_k(n+\frac{1}{2})$

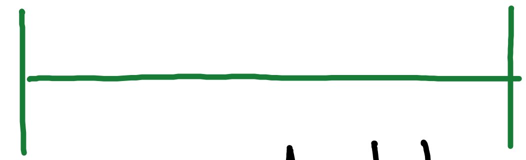
Full system has energy eigenstates

$$|n_1\rangle \otimes |n_2\rangle \otimes |n_3\rangle \otimes \ldots$$

with $$E = \sum_k \hbar\omega_k\left(n_k + \frac{1}{2}\right)$$

Interpretation:

$|0\ 0\ 0\ \ldots\rangle$



ground state

Interpretation:

$|1\ 0\ 0\ \ldots\rangle \longleftrightarrow$

ground state

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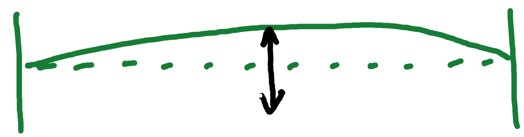
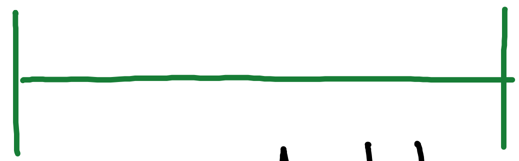


one quantum of longest wavelength mode

Interpretation:

$|0\ 0\ 0\ \ldots\rangle \longleftrightarrow$



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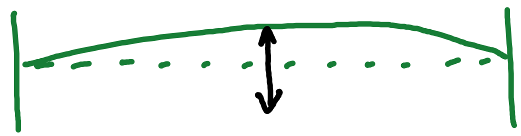
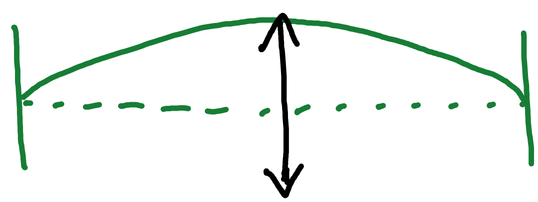


one quantum of longest wavelength mode

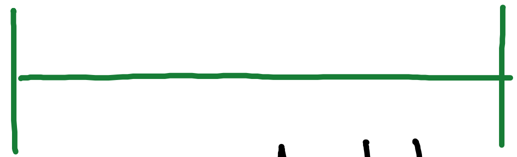
$|2\ 0\ 0\ \ldots\rangle \longleftrightarrow$



two quanta of longest wavelength mode

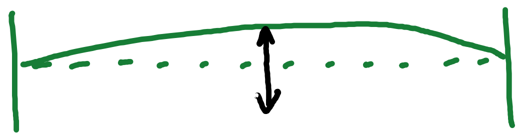
Interpretation:

$|0\,0\,0\ldots\rangle \longleftrightarrow$



ground state

$|1\,0\,0\ldots\rangle \longleftrightarrow$



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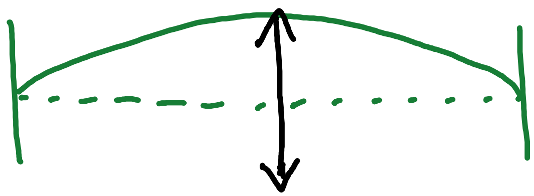
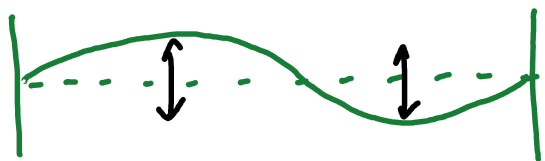


two quanta of longest wavelength mode

$|0\,1\,0\ldots\rangle \longleftrightarrow$



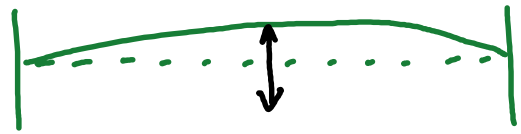
one quantum of 2nd longest wavelength mode

Interpretation:

$|0\,0\,0\ldots\rangle \longleftrightarrow$

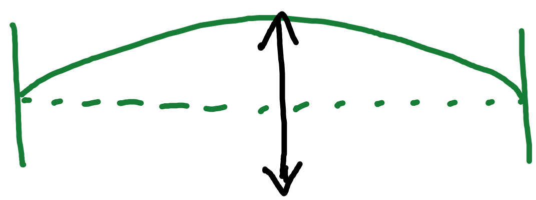
ground state

$|1\,0\,0\ldots\rangle \longleftrightarrow$



one quantum of longest wavelength mode $\equiv$ one PHOTON/phonon, etc... of $\lambda = 2L$

$|2\,0\,0\ldots\rangle \longleftrightarrow$



two quanta of longest wavelength mode $\equiv$ two PHOTONS of $\lambda = 2L$

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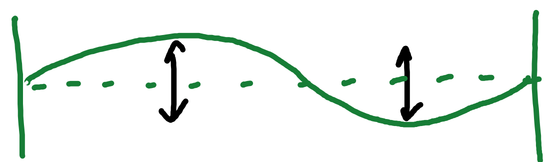


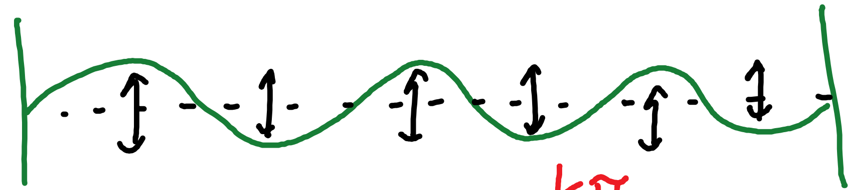
one quantum of 2nd longest wavelength mode

one PHOTON of $\lambda = L$

. . .

Wave energies are quantized

$$|0\;0\cdots 0\overset{k}{1}0\cdots\rangle$$

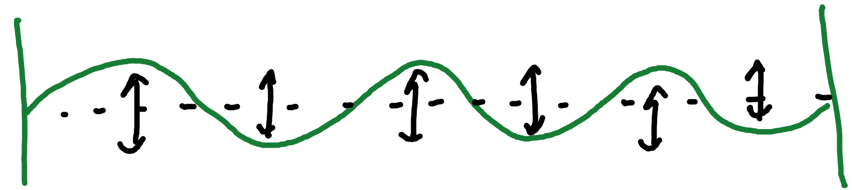


$$\omega_k = v \cdot \frac{k\pi}{L}$$

Q: what is the energy of one quantum of mode k?

Q: This mode has wavelength $\frac{2L}{k}$. What is the energy of a photon with this wavelength?

$$|0\;0\cdots 0\,\overset{k}{1}\,0\cdots\rangle$$



Check: energy of 1 quantum of mode k

$$E = \hbar\omega_k = \hbar\cdot v\,\frac{k\pi}{L}$$

wavelength is $\frac{2L}{k}$

Matches if v (wave speed) $= c$.

energy of 1 photon of wavelength $\frac{2L}{k}$

$$E = hf = h\frac{c}{\lambda} = h\cdot c\cdot\frac{k}{2L} = \hbar\cdot c\cdot\frac{k\pi}{L}$$

Summary so far: basis states of our field theory

= states with arbitrary number of PARTICLES
with each possible wavelength

$a_k^\dagger$ creates a particle w. wavelength $\frac{2L}{k}$
(i.e. momentum $\frac{hk}{2L}$)

all momenta possible for $L \to \infty$

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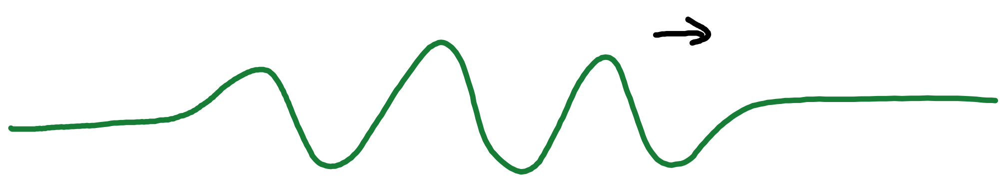
(i.e. momentum $\frac{hk}{2L}$)

Real photon: superposition of photons w. different wavelengths $\sum_k f(k) a_k^\dagger |0\rangle$



Warning: major spoiler on next slide. Stop watching if you are not ready for a significantly deeper understanding of the universe.

Amazing fact: This is how we understand ALL particles in nature!

EM field $\longrightarrow$ photons
$\vec{E}(x), \vec{B}(x)$

electron field $\longrightarrow$ electrons (and positrons!)
(NOT the wavefunction)
$\psi_\alpha(x)$

neutrino field
quark field
⋮
Higgs field

each obeys a wave equation

particles are the quanta of these waves.

Aside: why don't we have classical electron waves (like classical electromagnetic radiation?)

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Answer: electrons obey Pauli exclusion

classical EM wave $\longrightarrow$ lots of photons in same state

Pauli $\longrightarrow$ max 1 electron in each state, so can't build up classical wave.

THE STANDARD MODEL OF PARTICLE PHYSICS:

→ 1 field for each kind of particle

→ A wave equation [alternatively: a Lagrangian]

→ Quantum Hamiltonian is the energy operator for these fields.

Particle Interactions

Field theories describing interacting particles have wave equations w. non-linear terms, related to terms in the energy like $\lambda \int dx\, (h^4)$.

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$$\lambda \int dx \, (h^4).$$

$$\Big\Downarrow$$

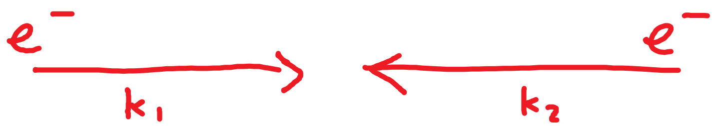
expand in Fourier modes, write w. creation and and annihilation operators

give terms in H like $a^\dagger_{k_1} a^\dagger_{k_2} a^\dagger_{k_3} a_{k_4}$

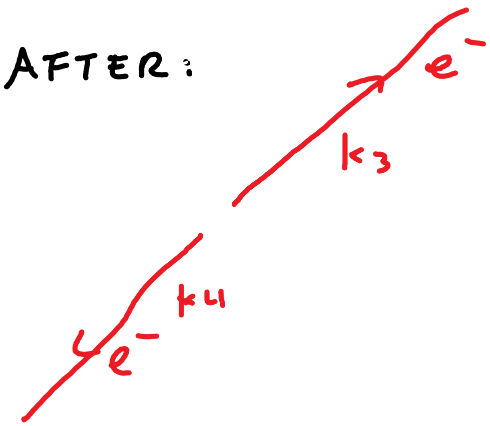
$$a^\dagger_{k_1} a^\dagger_{k_2} a_{k_3} a_{k_4}$$

Using time-dependent perturbation theory, these lead to transitions between states w. particles of different momenta or different numbers of particles:

BEFORE:

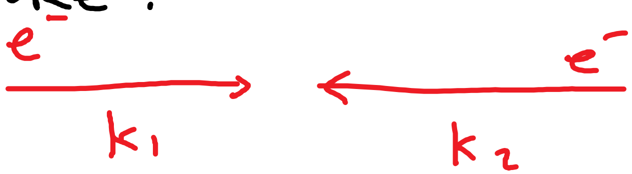


AFTER:



from: $\langle k_3 k_4 | a^\dagger_{k_3} a^\dagger_{k_4} a_{k_1} a_{k_2} | k_1 k_2 \rangle$

BEFORE:



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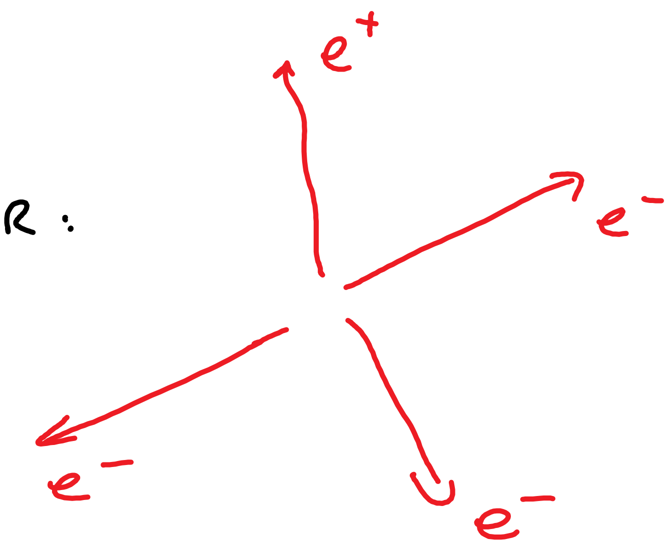

from $\langle \psi_f | a^\dagger a^\dagger a^\dagger a^\dagger a\, a | \psi_i \rangle$

Anything is possible as long as energy, momentum, charges are conserved.

Quantum field theory provides our most fundamental understanding of physics.