Short answer question (may be multiple choice on the test)

1. A particle moves in a potential with rotational symmetry about the z axis. What quantity is conserved?

2. Roughly how big is a hydrogen atom (in meters)?

3. What is the energy (in eV) of a photon released in the transition between $n = 2$ and $n = 1$ states of hydrogen?

4. What is $[\hat{x}, \hat{p}]$?

5. How can we tell if first order perturbation theory gives a reliable result for the energy shift of some state?

6. If $|\Psi_2\rangle$ is related to $|\Psi_1\rangle$ by an infinitesimal translation in the x direction, how can we express $|\Psi_2\rangle - |\Psi_1\rangle$ in terms of the original state $|\Psi_1\rangle$?

7. What is $\langle x|\hat{x}|y\rangle$ (i.e. evaluate this matrix element)?

8. Which of the following represents the probability of obtaining some particular value E in a measurement of energy on the state $|\Psi\rangle$?

   a) $\hat{H}|\Psi\rangle$

   b) $\langle\Psi|\hat{H}|\Psi\rangle$

   c) $\langle E|\Psi\rangle$

   d) $|\langle E|\Psi\rangle|^2$

   e) ΔE

9. If $\hat{\mathcal{O}}$ is a time-independent Hermitian operator and $\hat{H}$ is the time-independent Hamiltonian, which of the following is NOT a consequence of $[\hat{\mathcal{O}}, \hat{H}] = 0$?

   a) The expectation value of the physical observable associated with $\hat{\mathcal{O}}$ is unchanging in time.

   b) All states have definite values for both energy and the physical observable associated with $\hat{\mathcal{O}}$, since the matrices for both are diagonal.

   c) Given any solution $|\Psi(t)\rangle$ of the Schrodinger equation, $e^{ia\hat{\mathcal{O}}}|\Psi(t)\rangle$ is also a solution.

   d) The uncertainty in both $\mathcal{O}$ and $\mathcal{H}$ can be as small as we want at the same time.

   e) The probabilities for measuring various possible values of $\hat{\mathcal{O}}$ are unchanging in time.

10. For a general operator $\hat{B}$ without any special properties, which of the following is always equal to $\langle\chi|\hat{B}|\psi\rangle$?

    a) $\langle\psi|\hat{B}|\chi\rangle$

b) $\langle\psi|\hat{B}|\chi\rangle^*$

c) $\langle\psi|\hat{B}^\dagger|\chi\rangle$

d) $\langle\psi|\hat{B}^\dagger|\chi\rangle^*$

11. True or false: if $\langle\Psi|\hat{\mathcal{O}}|\Psi\rangle = 0$ for a Hermitian operator $\hat{\mathcal{O}}$ and for *all* states $|\Psi\rangle$ then it must be that $\hat{\mathcal{O}} = 0$.

    a) True

    b) False

    c) All of the above

    d) None of the above

12. If the state of a quantum system with a time-independent Hamiltonian is some energy eigenstate $|E\rangle$ at time $t = 0$, which of the following is NOT true

    a) The energy expectation value will be independent of time.

    b) All physical observables will be independent of time.

    c) If we measure the energy at any later time, we will always find E.

    d) The position space probability density for the state will oscillate periodically with a specific frequency.

13. If $\hat{\mathcal{O}}$ is an operator corresponding to some physical observable, we can say that

    a) There is a basis of states in the Hilbert space for which each basis element is an eigenstate of $\hat{\mathcal{O}}$ if and only if $\hat{\mathcal{O}}$ is a conserved quantity.

    b) There is a basis of states in the Hilbert space for which each basis element is an eigenstate of $\hat{\mathcal{O}}$ if and only if the transformation associated with $\hat{\mathcal{O}}$ is a symmetry.

    c) There is a basis of states in the Hilbert space for which each basis element is an eigenstate of $\hat{\mathcal{O}}$ if and only if the eigenvalues of $\hat{\mathcal{O}}$ are all different.

    d) There is always a basis of states in the Hilbert space for which each basis element is an eigenstate of $\hat{\mathcal{O}}$.

Written questions

1) A spin 1/2 particle is in the state $|S_x\rangle = \hbar/2$ at $t = 0$. A magnetic field $\vec{B} = B\hat{z}$ is turned on, and the S_x component of the spin is measured after time T. That is the probability that we will find $S_x = \hbar/2$.

2) Consider a 2 dimensional harmonic oscillator with Hamiltonian

$$\frac{p_x^2}{2m} + \frac{p_y^2}{2m} + \frac{1}{2}m\omega^2 x^2 + \frac{1}{2}m\omega^2 y^2 \tag{1}$$

a) Write expressions for all states in the second excited energy level.

b) What is the angular momentum operator for this system associated with rotations in the xy plane?

c) Write an expression for a state in the second energy level that is invariant under $x - y$ rotations.

2) A particle of spin 1/2 moves in a 1D harmonic oscillator potential with Hamiltonian

$$\frac{p^2}{2m} + \frac{1}{2}m\omega^2 x^2 + \omega S_z \tag{2}$$

a) What are the energy levels and degeneracies for this system?

b) A potential $H' = \alpha x S_x$ is added to the Hamiltonian. What is the change in the ground state energy to first order in perturbation theory?

c) What are the energy shifts for the states (or state) in the first excited energy level, again at first order in perturbation theory?

3) A particle of mass m moves in a 1D potential with a minimum at $x = 0$. The potential near $x = 0$ is approximately $V(x) = Qx^2/(1 - \lambda x)$.

a) What is the ground state energy for $\lambda = 0$?

b) How can we approximate the potential $V(x)$ near $x = 0$?

c) Write expressions in terms of matrix elements for the terms at first order and second order in perturbation theory that are proportional to λ^2.

4) Consider a system with two electrons at fixed positions (i.e. we can ignore their motion) in a magnetic field, so that the Hamiltonian is $H = A(S_z^1 + S_z^2)$.

a) Determine the energy levels and degeneracies for this system.

b) If we now add a perturbation $H_1 = cS_x^1 S_x^2$, determine the first order shift in the energy for each state.

5) Consider a system with two independent harmonic oscillators with frequency ω and 2ω (and the same mass m for each).

a) What is the Hamiltonian for this system?

b) For the lowest 5 distinct energy levels of the combined system, give the energy levels and degeneracies.

c) The oscillators are now coupled together by an interaction $H' = \lambda x_1^2 x_2$, where x_1 and x_2 are the coordinates for the oscillators with frequency ω and 2ω respectively. To first order in perturbation theory, what is the energy shift for the ground state?

d) What are the energy shifts at first order for the states in the second excited level and what are the corresponding eigenstates?

6) Consider a system with two independent harmonic oscillators each with frequency ω.

a) If we turn on an interaction between the particles that is described by a potential

$$V = \frac{1}{2}k(x_1 - x_2)^2 \tag{3}$$

calculate the new ground state energy to first order in perturbation theory.

b) For which values of k is your result reliable?

c) (BONUS) What are the exact energy levels for this system?

7) If the time-independent operator $\hat{\mathcal{O}}$ corresponding to some observable commutes with the Hamiltonian (also time-independent), show that the expectation value of $\mathcal{O}$ for any state is independent of time.