Problem Set 9

Question 0 (for Tuesday) Read the text in Griffiths chapter 7.1 carefully (Variational Method), and read through chapters 7.2 and 7.3. (probably ch 8 in new edition)

Question 1 Finish Thursday's worksheet before Tuesday's class.

Question 2 (Webwork) An electron in a hydrogen atom is in the state

$$|\Psi\rangle = \frac{1}{\sqrt{3}}|n=3, l=1, J=\frac{3}{2}, M=\frac{1}{2}\rangle + \sqrt{\frac{2}{3}}|n=2, l=0, J=\frac{1}{2}, M=-\frac{1}{2}\rangle \quad (1)$$

where J and M are the quantum numbers for total angular momentum J^2 and the z-component of angular momentum J_z.

a) Ignoring fine structure, what is the expectation value of energy in the state?

b) If we measure L_z (the z-component of orbital angular momentum) what are the possible values we might obtain and what are the probabilities of measuring them?

c) What is the expectation value of electron spin along the z direction?

d) What is the expectation value of total angular momentum J^2?

Question 3 (Webwork for Tuesday) a) Taking into account fine structure effects, how many different spectral lines will there be resulting from transitions between $n = 3$ states and $n = 2$ states in hydrogen?

b) Let ΔE_2 be the range of energies (highest minus lowest) for the $n = 2$ states of hydrogen, taking into account fine structure. What is $\Delta E_2/(E_2 - E_1)$, the ratio between this range and the energy difference between the $n = 2$ states and the $n = 1$ states of hydrogen (where $E_n = -13.6/n^2$)?

Question 4 (hand in Thursday) Consider a hydrogen atom (ignoring electron spin) in an electric field $\vec{E} = E\hat{z}$. Show that to first order in perturbation theory, the energy of the ground state does not change. Write an expression for the change in energy of the ground state to *second order* in perturbation theory, in terms of integrals involving the radial hydrogen wavefunctions $R_{nl}(r)$. To evaluate the angular integrals, you can use that $z = rcos(\theta) = rY_1^0(\theta,\phi)\sqrt{4\pi/3}$, together with the orthogonality relations (equation 4.33 in Griffiths) for the spherical harmonic functions Y_l^m.

Bonus points: Perform the integrals and sums to get a closed-form answer for the energy shift.