Problem Set 7

The main goal of this problem set is to give you practice with degenerate perturbation theory, the final topic that will be covered on the midterm. The online quiz in part 3 will also serve as a good review of the recent topics.

Problem 1

Consider a two-dimensional harmonic oscillator, with

$$H = \frac{p_x^2}{2m} + \frac{p_y^2}{2m} + \frac{1}{2}m\omega^2 x^2 + \frac{1}{2}m\omega^2 y^2 \tag{1}$$

a) What are the possible energies for this system and how many independent energy eigenstates are there at each energy? b) If we now add a perturbation

$$H \to H + \lambda H_1 \qquad\qquad H_1 = xy \tag{2}$$

find the lowest three energies up to order λ. *Hint: for $\lambda = 0$, two of these are the same, so you will need degenerate perturbation theory. Have a look at the "degenerate perturbation theory sample problem" on the website to see an example of the method being applied.* **Question 2** A particle of spin 1/2 confined in a certain quantum wire is governed by a Hamiltonian

$$H = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 x^2 + \omega S_z \tag{3}$$

with a harmonic oscillator part governing the position and an extra term involving the spin (perhaps due to a magnetic field). *Hint: you can think of this as a two-part quantum system where one part is a particle in a 1D harmonic oscillator, and the other part is a spin 1/2 system.*

a) What are the energy levels and degeneracies for this system?

b) A potential $H' = \alpha x S_x$ is added to the Hamiltonian. What is the change in the ground state energy to first order in perturbation theory? *Here, the x operator acts on the Harmonic oscillator part and the S_x operator acts on the spin part*

c) What are the energy shifts for the states (or state) in the first excited energy level, again at first order in perturbation theory?

Question 3 Prepare for and complete the (closed book) online multiple choice quiz covering perturbation theory and the harmonic oscillator before next class. You will have 15 minutes to complete the quiz, which you can find on Canvas. Practice quizzes are available on the course website.