Physics 402 Problem Set 12

This final problem set covers problems related to atomic transitions. It's specifically designed to prepare you for the types of problems that might appear on our exam. In fact, all of these problems did appear on exams, in 2017, 2018, and 2019.

Problem 0

Watch the April 2 videos.

Problem 1

A cavity contains $3N$ molecules with two available states. There are $2N$ molecules in the ground state $|0\rangle$ with energy E_0 and N molecules in an excited state $|1\rangle$ with energy $E_1 > E_0$. The cavity contains incoherent electromagnetic radiation; the energy density per unit frequency is described by some function $\rho(\omega)$.

a) Describe the various physical processes that could cause the number of molecules in each state to change with time.

b) If the matrix elements for the components of the electric dipole operator are given by

$$\langle 1|\mathcal{P}_x|0\rangle = \langle 1|\mathcal{P}_y|0\rangle = \langle 1|\mathcal{P}_z|0\rangle = p \ ,$$

what condition on $\rho(\omega)$ ensures that the number of molecules in each state will remain constant on average?

Problem 2

The temperature at the surface of the sun is about $5.8 \times 10^3 K$.

a) At this temperature, what is the ratio R_{stim}/R_{spon} between the rates for stimulated and spontaneous emissions from the $|210\rangle$ state of hydrogen to the ground state?

b) What is the spontaneous transition rate? *These may be useful:*

$$\psi_{100}(\vec{x}) = \frac{1}{\sqrt{\pi} a^{\frac{3}{2}}} e^{-\frac{r}{a}} \qquad \psi_{210}(\vec{x}) = \frac{1}{\sqrt{32\pi} a^{\frac{3}{2}}} \frac{r}{a} e^{-\frac{r}{2a}} \cos\theta \qquad (x,y,z) = (r\sin\theta\cos\phi, r\sin\theta\sin\phi, r\cos\theta)$$

$$\int_0^\infty ds s^4 e^{-s} = 24 \qquad \int_0^\pi d\theta \sin\theta \cos^2\theta = \frac{2}{3} \qquad \frac{(E_{100}/\hbar)^3 e^2}{3\pi\epsilon_0 \hbar c^3} = 9.47 \times 10^{29} m^{-2} s^{-1}$$

Problem 3

A particular molecule has three low-energy states $|\psi_1\rangle$, $|\psi_2\rangle$, and $|\psi_3\rangle$ with energies $-E_0$, $-E_0/2$, and $-E_0/4$ respectively. The matrix elements $\langle\psi_a|\vec{\mathcal{P}}|\psi_b\rangle$ of the electric dipole moment operator for these three states are

$$\mathcal{P}^x_{ab} = P_0 \begin{pmatrix} 1 & 2+i & 0 \\ 2-i & 3 & 1 \\ 0 & 1 & 2 \end{pmatrix} \qquad \mathcal{P}^y_{ab} = P_0 \begin{pmatrix} 0 & -i & 0 \\ i & 3 & 2 \\ 0 & 2 & 0 \end{pmatrix} \qquad \mathcal{P}^z_{ab} = P_0 \begin{pmatrix} 0 & -1 & 0 \\ -1 & 4 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

A collection of 100,000 of these molecules are prepared in the $|\psi_2\rangle$ state. The molecules are in an environment where the background radiation density is as shown in the plot.

What is the expected amount of time before only 99,900 molecules remain in the state $|\psi_2\rangle$? You may assume that only the three states described above are relevant and that processes involving multiple transitions (e.g. $2 \to 1 \to 2$) can be ignored.