Problem set 11

Question 1 (for Tuesday): Watch the March 26th videos linked on the course webpage. *Hint: you can try increasing the playback speed to make the videos shorter!* "Reading" question: write down a Hamiltonian for a charged spinless particle in a constant magnetic field $\vec{B} = B\hat{z}$, hand in on Canvas for participation credit.

Question 2 (Hand in Thursday on Canvas): Consider a one dimensional harmonic oscillator (mass m, frequency ω) in its ground state perturbed by

$$H'(t) = ax^2 sin(ft)$$

for $0 < t < \pi/f$, where a is a constant that is small compared to $m\omega^2$. What is the probability of measuring the oscillator in its first excited state after time $t = \pi/f$? What about for the second excited state? For the second excited state, plot this probability as a function of f/ω_0, where $\omega_0 = (E_2 - E_0)/\hbar$. For this problem, use the exact formula for the transition probability in first order perturbation theory (not the approximation specific to sinusoidal oscillations, since that is only reliable if the perturbation lasts for many oscillations). You can use a computer to graph the results for you.

Question 3 (Hand in Thursday on Canvas): A charged particle is in the ground state of an infinite square well potential with width a. At time $t = 0$, an electric field is applied so that the potential energy in the region $[0, a]$ becomes $V(x, t) = qE(t)x$, where $E(t)$ grows linearly with time until $t = T$ and then falls back to zero linearly with time. If the energy is measured at time $t = 2T$, what is the probability that the particle will be found in the first excited state?

b) There is a theorem in quantum mechanics called the adiabatic theorem. It states that if we are initially in the nth eigenstate of some Hamiltonian H0, and the Hamiltonian changes via some time dependent process to a final Hamiltonian H1 (which could be the same or different than H0), then in a limit where the Hamiltonian is changing slowly at all times, the final state will be the nth eigenstate of H1. Explain how your result is consistent with this theorem.